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Ensemble and Fuzzy Kalman Filter for position estimation of an autonomous underwater vehicle based on dynamical system of AUV motion

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ABSTRACT

An underwater vehicle is useful in the monitoring of the unstructured and dangerous underwater conditions. One of the unmanned underwater vehicle is AUV. AUV is a robotic device that is driven through the water by a propulsion system, controlled and piloted by an onboard computer, and maneuverable in three dimensions. This research explains about position estimation of AUV based on the Ensemble Kalman Filter (EnKF) and the Fuzzy Kalman Filter (FKF). EnKF is used as the estimation method of AUV's position that maneuvering in 6 DOF (Degrees of Freedom) with the specified trajectory. The estimation results are simulated with Matlab. The simulations show the AUV position estimation based on the EnKF with some of the different ensembles and the comparison results of the position estimation between the EnKF and the FKF. The final result of these study shows that Ensemble Kalman Filter is better to estimate the trajectory of the dynamical equation of AUV motion with the error estimation of EnKF is 92% smaller in the x-position dan y-position, 6.5% smaller in the z-position, 93% smaller in the angle dan the computation of time is 50% faster than the estimation results of FKF.

1. Introduction

Indonesia is a maritime country that has marine resources in the large quantities. The resources include species of flora, fauna, microbes, coral reefs, renewable resources or nonrenewable resources and others. That resources need a maintenance and monitoring regularly for safety of the country. Monitoring the condition of the unstructured and dangerous underwater needs an underwater vehicle that can overcome the condition. Vehicle that can be used for that monitoring is an unmanned underwater vehicle.

Unmanned underwater vehicle is being developed currently and it can be applied in several sector in life. That vehicle is important in many underwater activities because it has a high-speed, endurance and ability to dive more safely than humans (Yuh, 1994). One of the unmanned underwater vehicle is AUV (Autonomous Underwater Vehicle). AUV is a robotic device that driven through the water by a propulsion system, controlled and driven by the computer, and maneuverable in three dimensions (Von Alt, 2003).

A device that driven by a computer need an algorithm to guide and to control it's motion in the positioning, moreover the Global Positioning System (GPS) signal is not available underwater. Hence, Some innovative navigation strategy are designed for AUV. The Typhoon AUV is navigated based on the Unscented Kalman Filter (Allotta et al., 2016). The other algorithm is the Fuzzy Kalman Filter (FKF) that can be a positioning controller of AUV based on the determined trajectories (Ermayanti, Apriliani, Nurhadi, & Herlambang, 2015) and the FKF is also able to estimate performance parameters at off-nominal health conditions with fairly good accuracy (Rodger, 2012). Beside that, other algorithm is two stage rule based on the precision positioning control method for the linear piezoelectrically actuated table or LPAT (Kuo, Tarng, Nian, & Nurhadi, 2010). In CNC machine, one of the control method is TGPID (Nurhadi & Tarng, 2011). Beside a guidance and control, an optimization is needed in the positioning. Some research about optimization in the positioning is applied. In LPAT, a multistage rule based on the positioning optimization is applied to get a high precision LPAT (Nurhadi, 2011).

AUV is important in the underwater activities, so development of AUV should be done. One such development is the trajectory estimation of AUV. The estimation requires an appropriate method, such as a Data Assimilation. Data assimilation is an estimation

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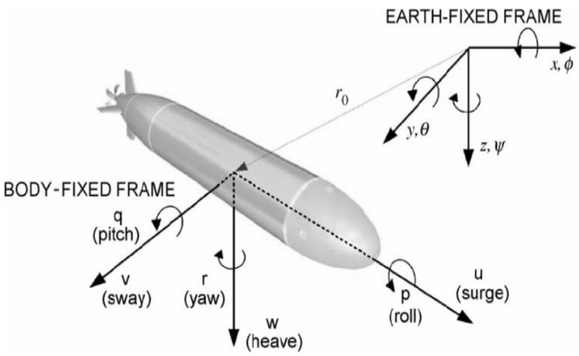

Fig. 1. Coordinate system of 6 DOF AUV (Yang, 2007).

method that combines mathematical models and measurement data (Lewis, Laksmivarahan, & Dhall, 2006). One of the data assimilation method is Kalman Filter. Kalman Filter is an estimation method for the linear dynamic stochastic system (Apriliani, Arif, & Sanjoyo, 2010). Some applications and modifications of Kalman Filter have been made to get a more accurate estimate and the computing time is shorter. Some modifications of the Kalman Filter are the ensemble Kalman filter (EnKF) and the Fuzzy Kalman Filter (FKF). EnKF is an estimation method for the non linear dynamic stochastic system based on measurement data, while the Fuzzy Kalman Filter is a modification of Kalman Filter that generate the variable state of system with the Fuzzy. EnKF is an effective method for estimation, one of that application is estimation of mobile robot position (Apriliani, Subchan Yunaini, & Hartini, 2013).

The authors developed a navigation and guidance control of AUV using the EnKF and FKF. The FKF estimation is the research before by Ermayanti et al. (2015) that gives the AUV position estimation based on the Fuzzy Kalman Filter. Each methods derive the estimation by using the 6 DOF (Degrees of Freedom) of the dynamical model of AUV that applied to the each estimation algorithm. The research before do a linearization to the dynamical model of AUV that applied in the FKF method, consequently the non linear model of AUV is become the linear form. In this occasion, we want to apply the non linear model of AUV using the EnKF to get the position estimation without any linearization like the research before. Therefore, we want to calculate the time computation and accuracy of the Enkf that compared with the FKF. This is simulated with Matlab and the Simulations show the AUV position estimation of each methods.

2. Dynamical model of AUV motion

There are two coordinate system needed which are used to describe the motion of AUV: earth-fixed (inertial) coordinates and body-fixed coordinates (Yang, 2007). Fig. 1 completely shows the definition of that coordinate system. Earth-fixed coordinates are used to describe the position and orientation of AUV. That coordinates are the x-axis that pointing north, the y-axis that pointing east and the z-axis that pointing towards the center of the earth. Body-fixed coordinates are used to describe the velocity and acceleration of the vehicles. Its origin is usually set at the center of gravity or the center of buoyancy, the x-axis is positive towards the bow, the y-axis is positive towards starboard, and the z-axis is positive downward. AUV that used in this research is AUV SEGOROGENI ITS. AUV SEGOROGENI ITS can be used for monitoring underwater and has specification in Table 1. AUV SEGOROGENI ITS uses only one propeller on the tail of AUV which will produce x_{prop} and additional moments K_{prop} (Herlambang, Djatmiko, & Nurhadi, 2015).

Table 1
AUV SEGOROGENI ITS specification.

Specification	Size
Weight	15 Kg
Overall length	980 mm
Beam	188 mm
Controller	Ardupilot Mega 2.0
Communication	Wireless Xbee 2.4 GHz
Camera	TTL Camera
Battery	Li-Po 11.8 v
Propulsion	12 V motor DC
Propeller	3 blades OD; 40 mm
Speed	1,94 knots (1 m/s)

Source: Ermayanti et al. (2015)

Table 2
The notation of AUV's equation of motion.

Motion	Forces and moments	Linear and angular velocities	Position and Euler angles
Surge (x-direction)	X	u	x
Sway (y-direction)	Y	v	y
Heave (z-direction)	Z	w	z
Roll (rotation about x)	K	p	ϕ
Pitch (rotation about y)	M	q	θ
Yaw (rotation about z)	N	r	ψ

The motion of AUV is 6 DOF, that is, three translations and three rotations along x, y, and z axis (Fossen, 1994). The general 6 DOF (Degrees of Freedom) motion equations of AUV are non linear system. That equations consist of surge, sway, heave, roll, pitch and yaw. Table 2 depicts the definition of the notation for AUV's equation of motion. That equation of AUV motion as follows (Yang, 2007):

- Surge

$$m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pq + \dot{q})] = X_{res} + X_{u|u|}u|u| + X_{\dot{u}}\dot{u} + X_{wq}wq + X_{qq}qq + X_{vr}vr + X_{rr}rr + X_{prop} \quad (1)$$

- Sway

$$m[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(pq + \dot{r})] = Y_{res} + Y_{v|v|}v|v| + Y_{r|r|}r|r| + Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{ur}ur + Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r \quad (2)$$

- Heave

$$m[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] = Z_{res} + Z_{w|w|}w|w| + Z_{q|q|}q|q| + Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{uq}uq + Z_{vp}vp + Z_{rp}rp + Z_{uw}uw + Z_{uu\delta_s}u^2\delta_s \quad (3)$$

- Roll

$$I_x\dot{p} + (I_z - I_y)qr + m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] = K_{res} + K_{p|p|}p|p| + K_{\dot{p}}\dot{p} + K_{prop} \quad (4)$$

- Pitch

$$I_y\dot{q} + (I_x - I_z)rp + m[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)] = M_{res} + M_{w|w|}w|w| + M_{q|q|}q|q| + M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{uq}uq + M_{vp}vp + M_{rp}rp + M_{uw}uw + M_{uu\delta_s}u^2\delta_s \quad (5)$$

- Yaw

$$I_z\dot{r} + (I_y - I_x)pq + m[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] = N_{res} + N_{v|v|}v|v| + N_{r|r|}r|r| + N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_{ur}ur + N_{wp}wp + N_{pq}pq + N_{uv}uv + N_{uu\delta_r}u^2\delta_r \quad (6)$$

The right hand side equations are external forces and moments. These forces and moments include hydrostatic force (buoyancy and weight), hydrodynamic force (drag and lift), added mass, control force (fin surface) and propeller thrust force.

The motion of AUV is described by two coordinates system, it is necessary to make clear the relationship of the transformation between that coordinates. Hence, we need to transform the linear and angular velocity of AUV in body-fixed coordinates to be the position and orientation in earth-fixed coordinates (Yang, 2007). The following equations define the transformation which is the kinematic model of the AUV (with the assumption of no motion in roll and pitch directions) (Ataei & Koma, 2014).

$$\dot{x} = u\cos(\psi) - v\sin(\psi) \quad (7)$$

$$\dot{y} = u\sin(\psi) + v\cos(\psi) \quad (8)$$

$$\dot{z} = w \quad (9)$$

$$\dot{\psi} = r \quad (10)$$

where $\dot{x}$, $\dot{y}$ and $\dot{z}$ are representation of speed in the Earth Fixed Frame (EFF) coordinate system, while u, v and w are representation of speed in the Body Fixed Frame (BFF) coordinate system and ψ is the yaw angle of the vehicle in EFF (Fig. 1).

3. Position estimation for AUV

The Estimation methods for the position estimation of AUV are the Ensemble Kalman Filter (EnKF) and the Fuzzy Kalman Filter (FKF). The model of AUV system for EnKF is the non linear equation, while the model of AUV system for FKF is the linear one. There is a linearization for the model of AUV system in the FKF method. Explanation of each method is in the following section.

3.1. Ensemble Kalman Filter

The Ensemble Kalman Filter is one of the Kalman Filter modifications (Evensen, 2003). The Ensemble Kalman Filter is an estimation method for the non linear dynamic stochastic system based on the measurement data. Firstly We derive the state space from the model of AUV motion Eqs. (1)–(6). The State space is used in the discretization steps. The steps of that as follow

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \frac{mz_G}{m-X_{\dot{u}}} & -\frac{my_G}{m-X_{\dot{u}}} \\ 0 & 1 & 0 & -\frac{mz_G}{m-Y_{\dot{v}}} & 0 & \frac{(mx_G-Y_{\dot{r}})}{m-Y_{\dot{v}}} \\ 0 & 0 & 1 & \frac{my_G}{m-Z_{\dot{w}}} & -\frac{(Z_{\dot{q}}+mx_G)}{m-Z_{\dot{w}}} & 0 \\ 0 & -\frac{mz_G}{I_x-K_{\dot{p}}} & \frac{my_G}{I_x-K_{\dot{p}}} & 1 & 0 & 0 \\ \frac{mz_G}{I_y-M_{\dot{q}}} & 0 & -\frac{(M_{\dot{w}}+x_G)}{I_y-M_{\dot{q}}} & 0 & 1 & 0 \\ -\frac{my_G}{I_z-N_{\dot{r}}} & \frac{(mx_G-N_{\dot{v}})}{I_z-N_{\dot{r}}} & 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} B1 \\ B2 \\ B3 \\ B4 \\ B5 \\ B6 \end{pmatrix} \quad (11)$$

Eq. (11) is a matrix that derived from the model of the AUV motion with B1, B2, B3, B4, B5 and B6 are non linear equation from the dynamical model of the AUV motion (Ngatini, Apriliani, & Nurhadi, 2016).

The general form of Eq. (11) can be written as

$$A\dot{V} = B \quad (12)$$

So the model system of AUV motion as follows

$$A\dot{V} = B$$
$$\dot{V} = A^{-1}B \quad (13)$$

Before we applied the Ensemble Kalman Filter to estimate the position of AUV, we discretize Eq. (13) respect to time, t, by using the Forward Finite Difference Method.

$$\dot{V} = \frac{V_{k+1} - V_k}{\Delta t} \quad (14)$$

The state space form of the model of the AUV motion is

$$V_{k+1} = f(V_k, k) \quad (15)$$

The model of AUV system is not exactly same with the real system, there is a noise system that cannot be written in the model. So that can be written as

$$V_{k+1} = f(V_k, k) + \varsigma_k \quad (16)$$

With $f(V_k, k)$ is the non linear function from AUV's equation of motion Eqs. (1)–(6), V_{k+1} is the state variable in time $k+1$, the velocity of AUV. ς_k is a noise system, which is a random vector drawn from Gaussian distribution with mean = 0 and covariance Q (Curn, 2014).

An observation equation is defined to make correlation between the state which we estimate and the observation model. The equation as follows:

$$Z_k = HV_k + \zeta_k \quad (17)$$

with Z_k is the observation. H is a matrix representing the observation model and ζ_k is the observation noise, which is a random vector drawn Gaussian distribution with mean = 0 and covariance R (Curn, 2014).

Suppose, we have a dynamic stochastic system Eq. (16) and an observation Eq. (17). By using Ensemble Kalman Filter method we estimate the state variables of Eqs. (1)–(6) with the observation data Eq. (17). Ensemble Kalman Filter is one of the modifications of the Kalman Filter method. Kalman Filter is the estimation method for the linear dynamic stochastic system, meanwhile Ensemble Kalman Filter is the estimation method for the non linear dynamic stochastic system based on the measurement (observation) data.

The algorithm of the Ensemble Kalman Filter is (Evensen, 2003)

a. The initial estimation
   Generate the n-ensembles of the initial estimation $V_{1,i} = [V_{1,1}V_{1,2}V_{1,3}\ldots V_{1,n}]$ with $V_{1,i} \sim N(\bar{V}_1, P_1)$.
   Mean of the initial estimation which generated : $\hat{V}_1 = \frac{1}{n}\sum_{i=1}^{N}\hat{V}_{1,i}$
b. The prediction step
   Generate N-ensemble for the variable state in the prediction step as follows

$$\hat{V}^-_{k+1,i} = f(\hat{V}_k, k) + \varsigma_{k,i} \quad (18)$$

   with $\varsigma_{k,\,i} \sim N(1, Q_k)$ is the ensemble of the noise system.
   Mean of prediction step estimation : $\hat{V}^-_{k+1} = \frac{1}{n}\sum_{i=1}^{N}\hat{V}^-_{k+1,i}$
   Error covariance of the prediction step estimation:
   $P^-_{k+1} = \frac{1}{n-1}\sum_{i=1}^{N}(\hat{V}^-_{k+1,i} - \hat{V}^-_{k+1})(\hat{V}^-_{k+1,i} - \hat{V}^-_{k+1})^T$
c. The correction step
   Generate the ensemble of the measurement data

$$Z_{k+1,i} = Z_{k+1} + \zeta_{k,i} \quad (19)$$

   with $\zeta_{k,\,i} \sim N(1, R_k)$ is the ensemble of measurement noise.
   Kalman gain is defined as $K_k = P^-_{k+1}H^T(HP^-_{k+1} + R_k)^{-1}$
   Estimation of the correction step is
   $\hat{V}_{k+1,i} = \hat{V}^-_{k+1,i} + K_k(Z_{k+1,i} - H\hat{V}^-_{k+1,i})$

Mean of the correction step estimation

$$\hat{V}_{k+1} = \frac{1}{n}\sum_{i=1}^{N} \hat{V}_{k+1,i} \tag{20}$$

with the error covariance is

$P_{k+1} = [1 - K_k H]P^{-}_{k+1}$

d. Substitute Eq. (20) into the prediction step Eq. (18).
e. Repeat and continue the step (b) and the step (c) until we get mean of the correction step estimation as the result of estimation.

The state of the system is composed of the velocity of the vehicle. We don't include position directly into the state vector, due to the initial condition is given at the time-k, while the estimation is calculated for the time-k+1. Hence, we need a linear and angular velocity transformation which is the kinematic model of the AUV (with the assumption of no motion in roll and pitch direction). It is shown in Eqs. (7)–(10). After we get the result of estimation in Eq. (20), $\hat{X}_k$, we transform the velocity into the position by using Eqs. (7)–(10).

3.2. Fuzzy Kalman Filter

Lotfi A. Zadeh is the person who first introduced the fuzzy set as a mathematical way to represent inaccuracies . If S is a collection of objects denoted by s, then the fuzzy set E is a set of sequential pairs that can be denoted as follows (Zadeh, 1965) :

$$E = \{(s, \mu_E(s)) \mid s \in S\} \tag{21}$$

Fuzzy Kalman Filter (FKF) is an estimation method that combines the fuzzy set with the Kalman Filter. We generate the variable state of AUV's velocity using the Fuzzy set, and then the Kalman Filter is applied to estimate that variable state. The variables are the linear velocities in surge, sway, and heave, and the angular velocities in roll, pitch and yaw. Characteristics of each of these variables have value within a certain range (e.g., the velocity in surge may be described as low or high) and these variables have many variability at different points in time. Fuzzy logics can help the researcher to write control statements to accommodate this variability (Rodger, 2012).

The research before by Ermayanti et al. (2015) established the Fuzzy Kalman Filter algorithm for AUV estimation that is explained bellow. Kalman Filter algorithm works in a linear system, so firstly we do linearization for the nonlinear system of AUV in Eq. (12) using Taylor Series. We do linearization for $A\dot{V} = B$ to get linear form $\dot{V} = CV + D\xi$, where $\xi = \begin{pmatrix} K_{prop} & \delta_s & \delta_r & X_{prop} & \delta_r & \delta_s \end{pmatrix}^T$.

$$C = A^{-1}J_V(B) \tag{22}$$

$$D = A^{-1}J_\xi(B) \tag{23}$$

Where A^{-1} is the matrix inversion of the matrix A, B^{-1} is the matrix inversion of the matrix B, $J_V(B)$ and $J_\xi(B)$ are the jacobian matrix B to the V and ξ, respectively. Then, we subtitute Eqs. (22) and (23) to the Eq. (24) to get the result of linearization.

$$\dot{V} = CV + D\xi \tag{24}$$

The linear model of Eq. (24) is controllable and observable (Herlambang, Djatmiko, & Nurhadi, 2016). We derive the system model from Eq. (24) that has been discretized using the finite difference method. The following system model is in Eq. (25).

$$\begin{pmatrix} u_{k+1}\\ v_{k+1}\\ w_{k+1}\\ p_{k+1}\\ q_{k+1}\\ r_{k+1}\end{pmatrix} = \begin{pmatrix} u_k\\ v_k\\ w_k\\ p_k\\ q_k\\ r_k\end{pmatrix} + \Delta t\left(C\begin{pmatrix} u_k\\ v_k\\ w_k\\ p_k\\ q_k\\ r_k\end{pmatrix} + D\begin{pmatrix} K_{prop}\\ \delta_s\\ \delta_r\\ X_{prop}\\ \delta_r\\ \delta_s\end{pmatrix}\right)$$

$$\begin{pmatrix} u_{k+1}\\ v_{k+1}\\ w_{k+1}\\ p_{k+1}\\ q_{k+1}\\ r_{k+1}\end{pmatrix} = (C\Delta t + 1)\begin{pmatrix} u_k\\ v_k\\ w_k\\ p_k\\ q_k\\ r_k\end{pmatrix} + D\Delta t\begin{pmatrix} K_{prop}\\ \delta_s\\ \delta_r\\ X_{prop}\\ \delta_r\\ \delta_s\end{pmatrix}$$

$$V_{k+1} = (C\Delta t + 1)V_k + D\Delta t\xi \tag{25}$$

We apply the system model Eq. (25) into the form of fuzzy. The fuzzy steps are the fuzzification, the basic rule fuzzy logic, the Fuzzy Kalman Filter algorithm and the defuzzification Chen, Xie, and Shieh (1998). The steps as follow.

a. Fuzzification
Fuzzification is a process of changing the input of the form crisp (firmly) into the form of fuzzy (variable linguistic) are presented in the form of sets of the fuzzy membership functions. It is generated by the fuzzy linear membership function. The initial value of the velocity is defined on the Table . The definition of each velocity is bellow.

$u \in [u^-, u^+]$
$v \in [v^-, v^+]$
$w \in [w^-, w^+]$
$p \in [p^-, p^+]$
$q \in [q^-, q^+]$
$r \in [r^-, r^+]$

The fuzzy linear membership function applies minimum and maximum values. Anything below the minimum is assigned a 0 (definitely not a member), while anything above the maximum is assigned a 1 (definitely a member). The fuzzy linear membership are consisted a positive slope linear fuzzy number and a negative slope linear fuzzy number. A positive slope is established when the maximum is greater than the minimum. If the minimum is greater than the maximum, a negative slope is established. A positive slope is established by increasingly the velocity or the maximum velocity is established, while a negative slope is established by decreasingly the velocity or the minimum velocity is established.
In this case, the membership function are bellow. We take an example for the surge velocity, u. The following membership function is applied for all the velocity.

1. When the velocity is decreasingly to be minimum

$$\mu_{u1} = \begin{cases} 1, & \text{when } u < u^-;\\ \frac{u-u^-}{u^+-u^-}, & \text{when } u^- \le u \le u^+;\\ 0, & \text{when } u^+ < u. \end{cases} \tag{26}$$

2. When the velocity is increasingly to be maximum

$$\mu_{u2} = \begin{cases} 1, & \text{when } u < u^+;\\ \frac{u^+-u}{u^+-u^-}, & \text{when } u^- \le u \le u^+;\\ 0, & \text{when } u^- < u. \end{cases} \tag{27}$$

b. Basic rule fuzzy logic
In general, the basic rule IF-THEN fuzzy logic is given as follows

$Rule^i : IF\ c\ is\ C_i\ then\ V^i_{k+1} = (\mu^i_C(c)\Delta t + 1)V_k + D\Delta t\xi$

Table 3
Initial value of the velocity.

Velocity	Explanation	Initial value
u^-	The minimum velocity surge	0
u^+	The maximum velocity surge	1
v^-	The minimum velocity sway	0
v^+	The maximum velocity sway	1
w^-	The minimum velocity heave	0
w^+	The maximum velocity heave	1
p^-	The minimum velocity roll	0
p^+	The maximum velocity roll	1
q^-	The minimum velocity pitch	0
q^+	The maximum velocity pitch	1
r^-	The minimum velocity yaw	0
r^+	The maximum velocity yaw	1

c is the initial value of the velocity (u, v, w, p, q, r), a 0 for minimum and a 1 for maximum. c is C_i means that c which is owned by the C has a membership value μ_C^i. Having established the basic rules, each rule is applied into the Kalman Filter algorithm, then the Kalman Filter algorithm generates the output filtering $\hat{V}_{k+1}^i$.
6 DOF AUV's equation of motion has 6 models or six variables. The state variables appear as much as 2^n or $2^6 = 64$. In general, form of the equation of a system is $V_{k+1} = (C\Delta t + 1)V_k + D\Delta t\xi$. As the existence of the basic rules , the form of the equation be $V_{k+1} = (C_i\Delta t + 1)V_k + D\Delta t\xi$. C_i is a matrix C with 64 possible (Table 4).

c. Fuzzy Kalman Filter Algorithm
The general form of the equation system on the Kalman Filter algorithm is $V_{k+1} = (C_i\Delta t + 1)V_k + D\Delta t\xi$. Matrix C and D are model of the system. In the algorithm of Fuzzy Kalman Filter, the matrix C on the system is changed to be the matrix C_i. Matrix C_i is derived from the basic rules of Fuzzy logic that has 64 rules with $C_k^i = \mu_C^i(c)c$. The algorithm of Fuzzy Kalman Filter as follows.

- System and measurement model

$$V_{k+1} = (C_k^i\Delta t + 1)V_k + D\Delta t\xi + \varsigma_k \tag{28}$$

$$Z_k = H_kV_k + \zeta_k$$
$$\varsigma_k \sim N(0, Q_k), \zeta_k \sim N(0, R_k) \tag{29}$$

- Initialization

$$\hat{V}(0) = \hat{V}_0$$
$$P(0) = P_0$$

- The prediction step

$$\hat{V}_{k+1}^- = (C_k^i\Delta t + 1)\hat{V}_k + D\Delta t\xi$$
$$P_{k+1}^- = C_k^iP_k(C_k^i)^T + Q$$

- The correction step
Kalman Gain:

$$K_{k+1} = P_{k+1}^-H_{k+1}^T(H_{k+1}P_{k+1}^-H_{k+1}^T + R_{k+1})^{-1}$$

Estimation of the correction estimation:

$$\hat{V}_{k+1} = \hat{V}_{k+1}^- + K_{k+1}(Z_{k+1} - H\hat{V}_{k+1}^-)$$

Error covariance:

$$P_{k+1} = (I - K_{k+1}H_{k+1})P_{k+1}^-$$

d. Defuzzification
Defuzzification is defined as the process of changing the amount shown in the form of the fuzzy sets of output fuzzy membership function to regain the form he asserted (crisp).

Table 4
Basic rule fuzzy logic.

Rule (i)	u	v	w	p	q	r	C_i	Rule (i)	u	v	w	p	q	r	C_i
1	1	1	1	1	1	1	C_1	33	0	1	1	1	1	1	C_{33}
2	1	1	1	1	1	0	C_2	34	0	1	1	1	1	0	C_{34}
3	1	1	1	1	0	1	C_3	35	0	1	1	1	0	1	C_{35}
4	1	1	1	1	0	0	C_4	36	0	1	1	1	0	0	C_{36}
5	1	1	1	0	1	1	C_5	37	0	1	1	0	1	1	C_{37}
6	1	1	1	0	1	0	C_6	38	0	1	1	0	1	0	C_{38}
7	1	1	1	0	0	1	C_7	39	0	1	1	0	0	1	C_{39}
8	1	1	1	0	0	0	C_8	40	0	1	1	0	0	0	C_{40}
9	1	1	0	1	1	1	C_9	41	0	1	0	1	1	1	C_{41}
10	1	1	0	1	1	0	C_{10}	42	0	1	0	1	1	0	C_{42}
11	1	1	0	1	0	1	C_{11}	43	0	1	0	1	0	1	C_{43}
12	1	1	0	1	0	0	C_{12}	44	0	1	0	1	0	0	C_{44}
13	1	1	0	0	1	1	C_{13}	45	0	1	0	0	1	1	C_{45}
14	1	1	0	0	1	0	C_{14}	46	0	1	0	0	1	0	C_{46}
15	1	1	0	0	0	1	C_{15}	47	0	1	0	0	0	1	C_{47}
16	1	1	0	0	0	0	C_{16}	48	0	1	0	0	0	0	C_{48}
17	1	0	1	1	1	1	C_{17}	49	0	0	1	1	1	1	C_{49}
18	1	0	1	1	1	0	C_{18}	50	0	0	1	1	1	0	C_{50}
19	1	0	1	1	0	1	C_{19}	51	0	0	1	1	0	1	C_{51}
20	1	0	1	1	0	0	C_{20}	52	0	0	1	1	0	0	C_{52}
21	1	0	1	0	1	1	C_{21}	53	0	0	1	0	1	1	C_{53}
22	1	0	1	0	1	0	C_{22}	54	0	0	1	0	1	0	C_{54}
23	1	0	1	0	0	1	C_{23}	55	0	0	1	0	0	1	C_{55}
24	1	0	1	0	0	0	C_{24}	56	0	0	1	0	0	0	C_{56}
25	1	0	0	1	1	1	C_{25}	57	0	0	0	1	1	1	C_{57}
26	1	0	0	1	1	0	C_{26}	58	0	0	0	1	1	0	C_{58}
27	1	0	0	1	0	1	C_{27}	59	0	0	0	1	0	1	C_{59}
28	1	0	0	1	0	0	C_{28}	60	0	0	0	1	0	0	C_{60}
29	1	0	0	0	1	1	C_{29}	61	0	0	0	0	1	1	C_{61}
30	1	0	0	0	1	0	C_{30}	62	0	0	0	0	1	0	C_{62}
31	1	0	0	0	0	1	C_{31}	63	0	0	0	0	0	1	C_{63}
32	1	0	0	0	0	0	C_{32}	64	0	0	0	0	0	0	C_{64}

The final result for Fuzzy Kalman Filter estimation is calculated by using the formula of average weight (Chen et al., 1998).

$$V_{k+1} = \frac{\rho^1 V_{k+1}^1 + \rho^2 V_{k+1}^2 + \rho^3 V_{k+1}^3 + \ldots + \rho^{64} V_{k+1}^{64}}{\rho^1 + \rho^2 + \rho^3 + \ldots + \rho^{64}} \quad (30)$$

The weight rho^i is defined by the user, the value of the input corresponding membership $(\rho^i = \mu_C^i(c))$. Deffuzification process produces crisp single estimate for each estimation. Where ρ is the weight. Values for each ρ is calculated as follows

$$\rho^1 = (\mu_{u^+}).(\mu_{v^+}).(\mu_{w^+}).(\mu_{p^+}).(\mu_{q^+}).(\mu_{r^+})$$
$$\rho^2 = (\mu_{u^+}).(\mu_{v^+}).(\mu_{w^+}).(\mu_{p^+}).(\mu_{q^+}).(\mu_{r^-})$$
$$\rho^3 = (\mu_{u^+}).(\mu_{v^+}).(\mu_{w^+}).(\mu_{p^+}).(\mu_{q^-}).(\mu_{r^+})$$
$$\rho^4 = (\mu_{u^+}).(\mu_{v^+}).(\mu_{w^+}).(\mu_{p^+}).(\mu_{q^-}).(\mu_{r^-})$$
$$\vdots$$
$$\rho^{64} = (\mu_{u^-}).(\mu_{v^-}).(\mu_{w^-}).(\mu_{p^-}).(\mu_{q^-}).(\mu_{r^-})$$

After we get the result estimation V_{k+1}, we need to transform that velocity into the position form using Eqs. (7–10). We compare the result estimation of EnKF dan FKF based on the error and the computational time of the estimation.

4. Simulation results

This section shows the result estimation of the EnKF and FKF algorithm. The model of the dynamical system of the AUV motion is the non linear model. We derive the state space based on that model. For the Ensemble Kalman Filter method, we use the non linear of the state space to be a model system but for the Fuzzy Kalman Filter method we linearizes the non linear state space into the linear form. That linear state space is a model system for the Fuzzy Kalman Filter method. We do linearization in the FKF because the FKF can compute the linear model. After we get the model system of each filter, we apply the algorithm of each Filter. That's why the XY and XZ executed paths are different for the two algorithms, but each of the model system is from the same model of the dynamical system of the AUV motion. In this research, we want to know what is the better algorithm, the EnKF with the original non linear model or the FKF with the linearization of the model. The chosen paths are derived based on the model system of each Filter. We just plot the graphs in x,y axis representing the turning path, and in x,z axis representing the diving path.

Each method has an initial velocity of zero. The rudder and stern angle are 5°. Value of the covariance matrix is 10^{-6}. Simulations in this research are in 2 dimension of graph. The graphs show the position of AUV in x, y and z axis based on the model of AUV motion 6 DOF. The paths of estimation are the real value of the estimation that derived from the system model of AUV motion. Eqs. (16) and (28) define the paths of the estimation based on the EnKF and FKF, respectively. The results of the estimation are derived from the average of 10 times estimation. The following sections define the result estimation of each method.

4.1. EnKF with some of different ensembles

This section shows the estimation result of EnKF with some of different ensembles. The selected ensembles are 50, 100, 200 and 300. We simulated the estimation algorithm for 10 times and averaged that simulation in to the result of estimation. Table 5 represents the accuracy and the computational time of each ensembles. Table 5 shows that EnKF estimates the AUV position accurately. EnKF with 200th ensemble is the most accurate to estimates the X position and Z position, EnKF with 300th ensemble give the best estimation for the Y position, and the most accurate for angle is given by 50th Ensemble and 300th ensemble. The most accurate estimation is given by the different ensemble for each position, but it is estimated exactly by each ensembles. Table 5 also shows the computational time of each ensemble. The shortest time is given by 50th ensemble. More ensemble need more time computational. 50th ensemble gives exactly estimation with the shortest time computational.

Table 5
RMSE and computational time of EnKF.

Ensemble	RMSE (m/rad)				Time (s)
	X Position	Y Position	Z Position	Angle	
50	0,0015	0,0014	0,00112	0,000027	1,69
100	0,0022	0,0017	0,0010	0,000029	3,23
200	0,0014	0,0011	0,00064	0,000028	6,50
300	0,0026	0,00077	0,0011	0,000027	10,12

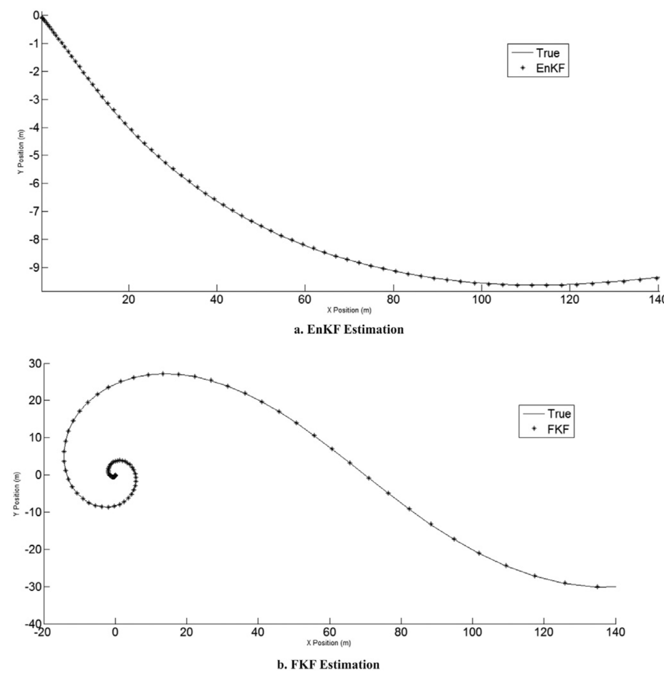

Fig. 2. XY Position estimation.

4.2. Ensemble and Fuzzy Kalman Filter

Fig. 2 shows the estimation in XY position or estimation in Surge and Sway motion. Fig. 3 shows estimation in XZ position or estimation in Surge and Heave motion. Each figure compare the estimation results of the EnKF and FKF for AUV position. Figs. 2 and 3 show that each method has good performance in estimation. Numerical indices are calculated in Table 3 to represent the accuracy and computational time of EnKF algorithm compared to FKF.

The root mean square error (RMSE) of the AUV position are compared in Table 6 to know accuracy of each method. The RMSE from EnKF is smaller than the RMSE from FKF. In addition, the computational time of EnKF is less than FKF. Therefore, the EnKF estimation is better than the FKF estimation.

As it is shown in Table 7, the error estimation in final position is calculated. The EnKF estimation is closer with the trajectory of AUV than the FKF estimation. The error percentage of EnKF in final position is less than 8%, while the error percentage of FKF is more than 8%. That errors percentage are shown in Table 7.

Table 6
The RMSE of the AUV position.

Method	X Position (m)	Y Position (m)	Z Position (m)	Angle (rad)	Time (s)
EnKF	0,00152	0,00138	0,001123	0,000027	1688380
FKF	0.01981	0.01778	0.002287	0.000376	3.3382

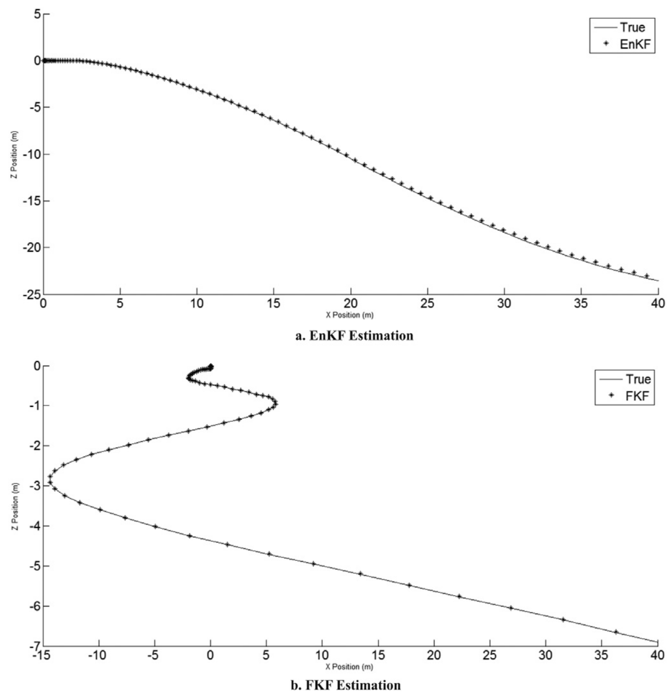

Fig. 3. XZ Position estimation.

Table 7
The error percentage of estimation in final position.

Method	X Position (%)	Y Position (%)	Z Position (%)
EnKF	0,032	0,051	0,0768
FKF	0,16	0,52	0,085

The final result of these study shows that Ensemble Kalman Filter is better to estimate the trajectory of the dynamical equation of AUV motion with the error estimation of EnKF is 92% smaller in x-position dan y-position, 6.5% smaller in z-position, 93% smaller in angle dan the computation of time is 50% faster than the estimation results of FKF.

5. Conclusion

In this paper, the positions of AUV are estimated. The values of the dynamical system model of AUV motion from each method are used as the true values or paths and then compared with the estimated parameters. We can apply the Ensemble Kalman Filter (EnKF) and Fuzzy Kalman Filter (FKF) to estimate the position of AUV based on the dynamical system model of AUV motion. The performance of each method is compared based on the accuracy and computational time of estimation. The error estimation of EnKF is 92% smaller in x-position dan y-position, 6.5% smaller in z-position, 93% smaller in angle dan the computation of time is 50% faster than the estimation results of FKF. RMSE of EnKF is smaller than RMSE of FKF and computational time of EnKF is less than FKF. Hence, the Ensemble Kalman Filter is better than the Fuzzy Kalman Filter to estimate AUV's position in this case.

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