Steady-State Cornering Modeling and Analysis of Three-Wheel Narrow Vehicle

Unggul Wasiwitono[1,a], I Nyoman Sutantra[1,b], Yohanes[1,c], and Yunarko Triwinarno[2,d]

[1]Otomotive Laboratory, Mechanical Engineering Department, Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia

[2]Vibration and Dynamics System Laboratory, Mechanical Engineering Department, Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia

[a]unggul@me.its.ac.id, [b]tantra@me.its.ac.id, [c]yunus@me.its.ac.id, [d]yunarko@me.its.ac.id

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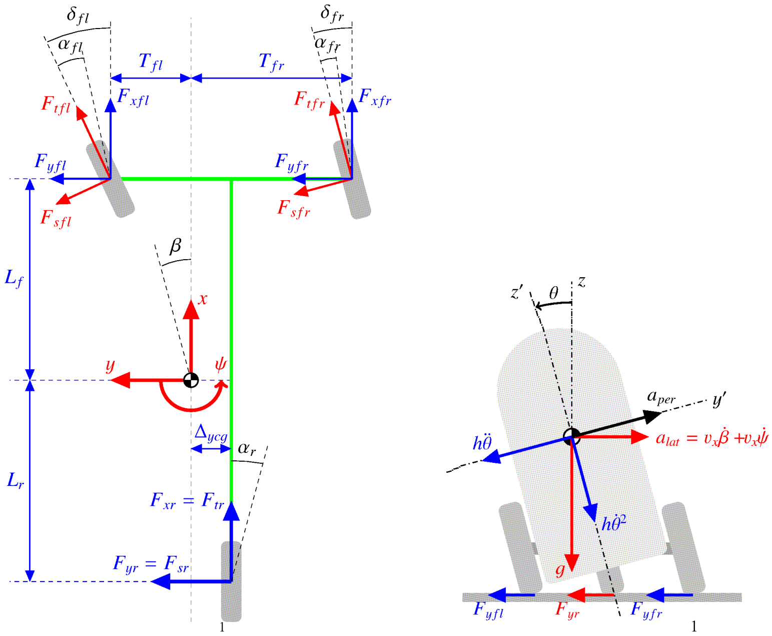
Abstract. Electric mobility seems to be an innovative alternative to future urban transport. In this study, a steady-state cornering model of a three-wheel narrow electric vehicle is derived. The steady-state cornering analysis is conducted by varying the location of the vehicle center of gravity, speed and tilt angle. From this analysis, the center of gravity location and tilt angle that gives better cornering characteristics can be obtained. Therefore, this analysis helps and can be used as starting point to design the chassis and the tilting control system of the three-wheel narrow electric vehicle.

Introduction

In order to address way to cater with the limited fossil fuel availability, electric mobility is one of several alternative to future transport systems. Promoting electric vehicles could offer a locally emissions-free and quiet transport system. Recently, a diverse range of new electric vehicles with better energy efficiency and a new driving system are designed by several auto manufacturers. In addition, in terms of the space and weight savings, narrow vehicles could offers a solution to congestion problems experienced in urban environments. Therefore, narrow electric vehicle seem to be an innovative alternative to future urban transport systems.

Narrow vehicles have been extensively studied in the last decade. The modeling and control system for a three-wheeled narrow vehicle with one front and two rear wheels are given in [1,2], with two front and one rear wheels are given in [3] and with four wheels are given in [4]. A direct control of the perceived acceleration which is the measured resultant acceleration at the vehicle center of gravity is usually used as a tilt control of the narrow vehicle [4-6]. Three tilt control system are available, i.e., direct tilt control, steering tilt control and the combination of the aforementioned tilt control systems. The performance of different tilt control system on the lateral stability is studied in [5-6]. A study on a sliding mode controlled three-wheel narrow vehicle for two passengers is reported in [7]. In addition, the study of in wheel motor that is used to propel the two rear-driven three-wheel narrow electric vehicle is conducted in [7,8].

A key problem in building up a traffic system using a narrow vehicle is how to develop a stable and easily maneuverable narrow vehicle. An alternative solution to this problem is by making the narrow vehicle have an ability to tilt into corners like a motorcycle. Although many modeling and control strategy have been proposed for the narrow tilting vehicle (NTV), to the best of the author's knowledge, the steady-state cornering of NTV hasn't got much attention. By understanding the narrow vehicle characteristics in steady-state cornering, the fundamental characteristic of narrow vehicle motion can be understood. The work described in this paper is an initial part of an effort to develop a three-wheeled tilting narrow electric vehicle for the urban environment. The steady-state cornering behavior analysis is conducted with varying the location of the vehicle center of gravity, speed and tilt angle. From this analysis, the center of gravity location and tilt angle that gives better cornering characteristics can be obtained.



(a) (b)

Fig. 1: Forces acting on three-wheel vehicle

Vehicle Modeling

The steady turning behavior at constant forward speed due to a small and steady steer angle is considered in this study. Fig. 1 shows the schematic and force acting on three-wheel vehicle. The side forces acting on each wheel are depending on the normal force and friction coefficient at each wheel that is related to slip angle α. Although the relationship between the side forces and slip angles is nonlinear, for small slip angles, we can use the following approximation for forces acting in y direction at each wheel:

$$F_{yfl} = C_{fl}\alpha_{fl} = C_{fl}\left(-\beta - \frac{L_f}{v_x}\dot{\psi} + \delta_{fl}\right) \tag{1}$$

$$F_{yfr} = C_{fr}\alpha_{fr} = C_{fr}\left(-\beta - \frac{L_f}{v_x}\dot{\psi} + \delta_{fr}\right) \tag{2}$$

$$F_{yr} = C_r\alpha_r = C_r\left(-\beta + \frac{L_r}{v_x}\dot{\psi}\right) \tag{3}$$

where C_{fl}, C_{fr} and C_r are the tire cornering stiffness, β is the side-slip angle, $\dot{\psi}$ is yaw rate, L_f is the distance from vehicle center of gravity to the front axle, L_r is the distance from vehicle center of gravity to the rear axle, v_x is the forward vehicle velocity, δ_{fl} and δ_{fr} are the left and the right front wheel steering angle.

First, let consider Fig. 1a. Applying the Newton's second law to translation on y direction gives

$$m_c\left(v_x\dot{\beta} + v_x\dot{\psi}\right) = F_{yfl} + F_{yfr} + F_{yr} \tag{4}$$

where m_c is the vehicle mass. In this case, the acceleration in the y direction is due to the centripetal acceleration of a vehicle in a steady turn. In addition, the sum of moments about the center of gravity to be zero in the steady turn, this gives

$$I_{zz}\ddot{\psi} = L_f\left(F_{yfl} + F_{yfr}\right) - L_rF_{yr} - T_{fl}F_{xfl} + T_{fr}F_{xfr} + \Delta_{ycg}F_{xr} \tag{5}$$

where T_{fl} is the distance from vehicle center of gravity to the left wheel, T_{fr} is the distance from vehicle center of gravity to the right wheel, Δ_{ycg} is the distance from vehicle center of gravity to the rear wheel on the y axis, F_{xfl}, F_{xfr}, and F_{xr} are forces acting in x direction at each wheel as shown in Fig. 1a. Further, define $C_f = C_{fl} + C_{fr}$ and $\delta_f = \frac{\delta_{fl}+\delta_{fr}}{2}$, we have

$$F_{yf} = F_{yfl} + F_{yfr} = C_f\alpha_f = C_f(-\beta - \frac{L_f}{v_x}\dot{\psi} + \delta_f) \tag{6}$$

To simplify the analysis, first we consider the zero tilting angle and assume $\Delta_{ycg} = 0$. Substituting Eqs (3) and (6) into Eqs (4) and (5) then rearrange, we have

$$\begin{aligned} m_c v_x\dot{\beta} + (C_f + C_r)\beta + (m_c v_x + \frac{C_f L_f}{v_x} - \frac{C_r L_r}{v_x})\dot{\psi} &= C_f\delta_f & (7)\\ (L_f C_f - L_r C_r)\beta + I_z\ddot{\psi} + (\frac{C_f L_f^2}{v_x} + \frac{C_r L_r^2}{v_x})\dot{\psi} &= L_f C_f \delta_f & (8) \end{aligned}$$

The above equations are the fundamental equation of motion describing the vehicle motion characteristic in response to a steady front wheel steer angle δ_f.

The steady-state cornering is a condition where the vehicle moving at a constant speed and fixed front steer angle. Under such conditions the vehicle will make a steady circular motion with constant turning radius. During steady-state cornering the side-slip angle and the yaw velocity are not changes, in this case $\ddot{\psi} = 0$, $\dot{\beta} = 0$. Therefore, Eqs (7) and (8) can be written as

$$\begin{aligned} (C_f + C_r)\beta + \left(m_c v_x + \frac{C_f L_f}{v_x} - \frac{C_r L_r}{v_x}\right)\dot{\psi} &= C_f\delta_f & (9)\\ (L_f C_f - L_r C_r)\beta + \left(\frac{C_f L_f^2}{v_x} + \frac{C_r L_r^2}{v_x}\right)\dot{\psi} &= L_f C_f\delta_f & (10) \end{aligned}$$

Expanding and rearranging Eqs (9) and (10), we obtain the solution for $\dot{\psi}$ for an arbitrary front wheel steer angle

$$\dot{\psi} = \frac{v_x(-L_r - L_f)C_f C_r}{m_c v_x^2 L_f C_f - m_c v_x^2 L_r C_r + \left(-2L_f L_r - L_f^2 - L_r^2\right)C_f C_r} \tag{11}$$

If the forward velocity of the vehicle is constant, the actual turning radius can be approximate as

$$\rho = \frac{v_x}{\dot{\psi}} = \left(\frac{m_c v_x^2 L_f C_f - m_c v_x^2 L_r C_r + \left(-2L_f L_r - L_f^2 - L_r^2\right)C_f C_r}{v_x(-L_r - L_f)C_f C_r}\right)\frac{1}{\delta_f} \tag{12}$$

Considering the geometry of the narrow vehicle, the roll stability is an important issue. To improve the roll stability and reducing risk, this vehicle should have the ability to tilt into corners like a motorcycle, as shown in Fig. 1b. The tilt angle that stabilize the vehicle is depend on the vehicle speed, yaw rate and road bank angle. For the current study, zero road bank angle is assumed. Applying the Newton's second law gives

$$\begin{aligned} m_c a_y &= F_{yfl} + F_{yfr} + F_{yr} & (13)\\ I_z\ddot{\psi} &= L_f(F_{yfl} + F_{yfr}) - L_r F_{yr} & (14)\\ I_x\ddot{\theta} &= m_c g h\sin\theta - m_c h^2\ddot{\theta}\sin^2\theta - m_c h\dot{\theta}^2\cos\theta\sin\theta - (F_{yfl} + F_{yfr} + F_{yr})h\cos\theta & (15) \end{aligned}$$

where $a_y = v_x\dot{\beta} + v_x\dot{\psi} + h\ddot{\theta}\cos\theta - h\dot{\theta}^2\sin\theta$, h is the position of the center of gravity on the z' axis, and g is the gravitational constant.

Linearize the above nonlinear model around $\theta = 0$ and substituting the Eqs (3) and (6) into Eqs (13) – (15), the nonlinear model (13) – (15) can be simplify to the following linear model

$$m_c\left(v_x\dot{\beta}+v_x\dot{\psi}+h\ddot{\theta}\right) = C_f(-\beta-\frac{L_f}{v_x}\dot{\psi}+\delta_f)+C_r\left(-\beta+\frac{L_r}{v_x}\dot{\psi}\right) \quad (16)$$

$$I_z\ddot{\psi} = L_fC_f(-\beta-\frac{L_f}{v_x}\dot{\psi}+\delta_f)-L_rC_r\left(-\beta+\frac{L_r}{v_x}\dot{\psi}\right) \quad (17)$$

$$I_x\ddot{\theta} = m_cgh\theta-\left(C_f(-\beta-\frac{L_f}{v_x}\dot{\psi}+\delta_f)+C_r\left(-\beta+\frac{L_r}{v_x}\dot{\psi}\right)\right)h \quad (18)$$

Substituting $\ddot{\theta}$ from Eq (18) into Eq (16) and considering steady-state condition, we have

$$\frac{m_cv_x^2+a_1\left(1+m_ch^2\right)}{v_x}\dot{\psi}+\frac{I_xa_2+a_2m_ch^2}{I_x}\beta = \left(C_f+\frac{C_fm_ch^2}{I_x}\right)\delta_f-\frac{m_c^2gh^2}{I_x}\theta \quad (19)$$

$$\frac{L_f^2C_f+L_r^2C_r}{v_x}\dot{\psi}+a_1\beta = L_fC_f\delta_f \quad (20)$$

where $a_1 = L_fC_f - L_rC_r$ and $a_2 = C_f + C_r$. The above equations are the fundamental equation of motion describing the narrow vehicle motion characteristic in response to a steady front wheel steer angle δ_f and tilt angle θ. Expanding and rearranging Eqs (19) and (20), we obtain the solution for $\dot{\psi}$ for an arbitrary steer angle and tilting angle

$$\dot{\psi} = \frac{\left(-I_xL_rC_fC_r-I_xL_fC_fC_r+2L_fC_f^2m_ch^2\right)v_x\delta_f-m_c^2gh^2v_x\left(L_fC_f-L_rC_r\right)\theta}{m_cv_x^2L_fC_fI_x-m_cv_x^2L_rC_rI_x+a_3I_x+a_3m_ch^2} \quad (21)$$

where $a_3 = 2L_f^2C_f^2+\left(L_f^2+L_r^2-2L_fL_r\right)C_fC_r+2L_r^2C_r^2$.

The vehicle stability is assured if the sum of the forces acting at the center of the gravity is zero. Therefore, to assure the stability of the vehicle, the resultant acceleration at the center of gravity a_{res} (perceived acceleration) define in the following equation must be zero,

$$a_{res} = \left(v_x\dot{\beta}+v_x\dot{\psi}\right)\cos\theta+h\ddot{\theta}-g\sin\theta \quad (22)$$

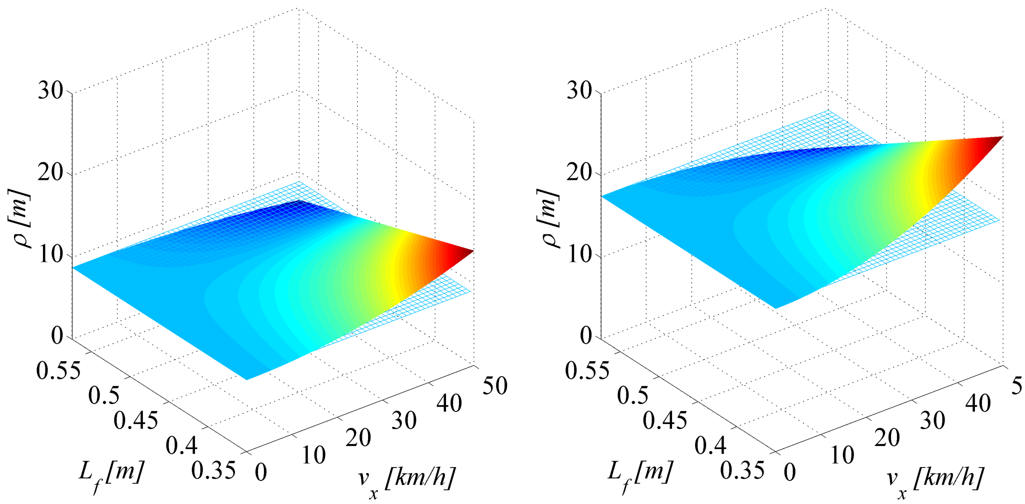
Further, for steady-state condition the resultant acceleration at the center of gravity can be written as

$$a_{res} = v_x\dot{\psi}\cos\theta-g\sin\theta \quad (23)$$

Numerical Simulation

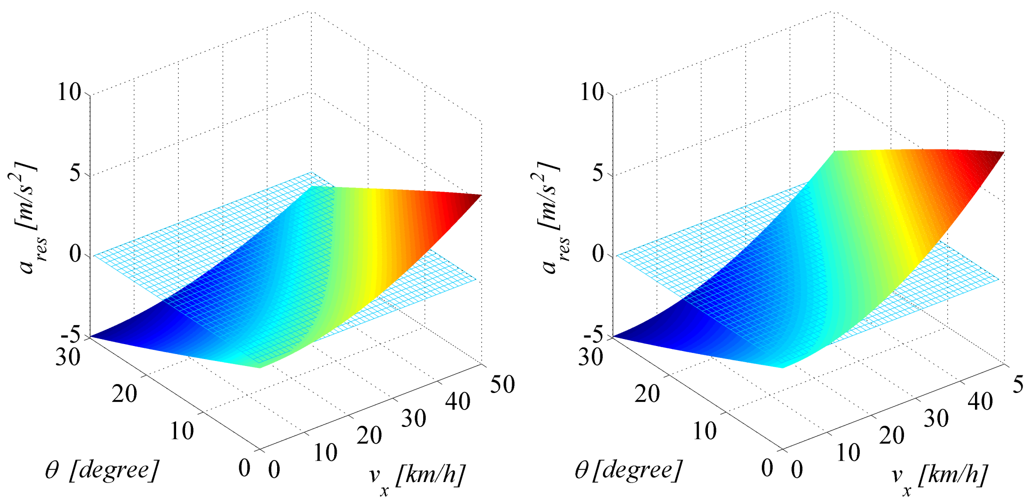
In order to study the steady-state cornering behavior of the three-wheel narrow vehicle, the parameters of the NTV used in [6] are taken as initial design parameters. Next, the steady-state cornering behavior of the three-wheel narrow vehicle is studied by comparing the actual turning radius for increasing vehicle speed based on Eq (12) with the Ackermann turning radius. Fig. 2 shows the actual turning radius for front wheel steer angle 10^0 and 5^0. These steer angles are associated with Ackermann turning radius 8.709 m and 17.418 m, respectively. As expected, it can be seen from the figure that moving the center of gravity location away from the front axle will decrease the vehicle turning radius as vehicle speed is increased. For $L_f > 0.5$, the actual turning radius is getting smaller than the Ackerman turning radius for increasing vehicle speed. It's mean that the vehicle will have an over-steer characteristic. On the other hand, when $L_f < 0.5$, the vehicle will have an under-steer characteristic.

Let now study the effect of tilting angle on the perceived acceleration. Fig. 3 shows the perceived acceleration as a function of tilting angle and vehicle speed for the case $L_f = 0.5$ m and for the position of the center of gravity on the z' axis 0.8 m and 1 m, respectively. It can be seen from the figure that the tilting angle which gives zero perceived acceleration depends on the vehicle speed and significantly



(a) Ackermann turning radius 8.709 m (b) Ackermann turning radius 17.418 m

Fig. 2: Turning radius as a function of center of gravity location and vehicle speed



(a) $h = 0.8$ m (b) $h = 1$ m

Fig. 3: Resultant acceleration at the center of gravity a_{res} as a function of tilting angle and vehicle speed for $L_f = 0.5$ m.

influenced by the height of the center of gravity. Lowering the position of the center of gravity by 0.2 m can significantly reduce the tilting angle that give zero perceived acceleration of the same vehicle speed. In addition, large tilting angle at low speed will gives negative resultant acceleration at the center of gravity.

The effect of the center of gravity location (L_f) and the tilt angle on the perceived acceleration for the case of vehicle speed 40 km/h is shown in Fig. 4. From that figure it is clear that moving the center of gravity away from the front axle will increase the tilt angle required to achieve zero perceived acceleration. Furthermore, it is also clear from the figure that for the same tilt angle, moving the center of gravity away from the front axle will increase the magnitude of the perceived acceleration.

Summary

The steady-stated cornering behavior of three-wheel narrow vehicle is studied. The steady-state cornering behavior analysis is conducted with varying the location of the vehicle center of gravity, speed and cornering radius. The center of gravity location that will give better cornering and steer charac-

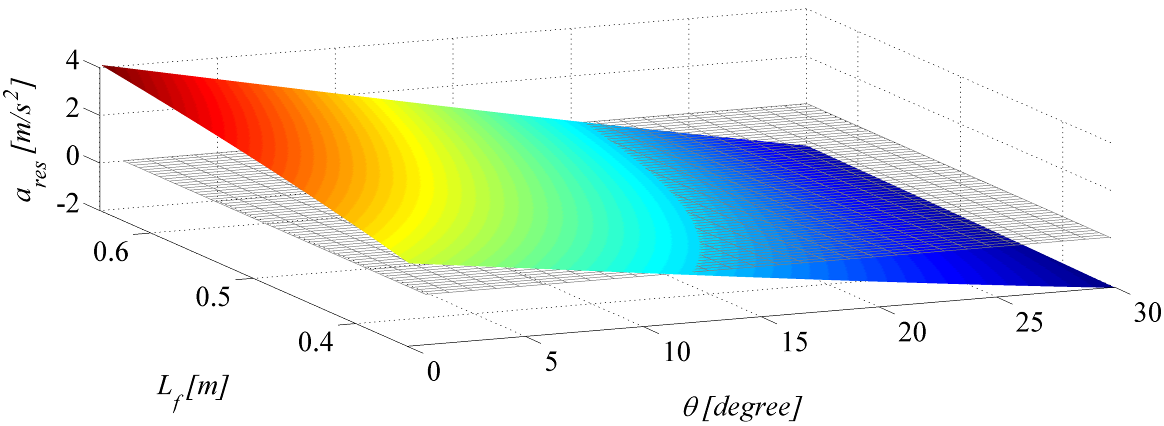


Fig. 4: Resultant acceleration at the center of gravity a_{res} as a function of tilting angle and center of gravity location (L_f) for vehicle speed 40 km/h and $h = 0.8$ m.

teristic is around 0.5 m from the front axle which is one third of the wheel base length. Lowering the position of the center of gravity resulting in a smaller tilting angle needed to achieve zero perceived lateral acceleration. Further, this analysis helps and can be used as starting point to design the chassis and the tilting control system of the three-wheel narrow electric vehicle.

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