COLOR EDGE DETECTION USING THE MINIMAL SPANNING TREE ALGORITHM AND VECTOR ORDER STATISTIC

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Abstract
The edge detection approach based on minimal spanning tree and vector order statistic is proposed. Minimal spanning tree determined ranking from the observations and identified classes that have similarities. Vector Order Statistic view a color image as a vector field and employ as a distance metrics. Experiment of edge detection on several images show that the result of minimal spanning tree is smoother and more computational time comparing to that vector order statistic.

Keywords: edge detection, Minimal Spanning Tree, Vector Order Statistics.

1. Introduction
Edge detection is a very important low-level vision operation. Despite the fact that a great number of edge detection methods have been proposed in the literature so far, there is still a continuing research effort. Recently, the main interest has been directed toward algorithms applied to color [1] and multispectral images [6], which also have the ability to detect specific edge patterns like corners and junctions [2].

Edges are defined, in digital image processing terms, as places where a strong intensity change occurs. Edge detection techniques are often required in different tasks in image processing and computer vision applied to areas such as remote sensing or medicine, to preserve important structural properties, image segmentation, pattern recognition, etc [7]. Another method to edge detection is using YUV Space and Minimal Spanning Tree [8].

Scalar order statistics have played an important role in the design of robust signal analysis techniques. Statistic ordering can be easily adapted to unvaried data, but for multivariate data, it must go through preprocessing before it can be ordered. For this Vector Order Statistic method, R-ordering is used because based of test result; it is the best method to be used on color image processing.

In this work, a new approach for ordering and clustering multivariate data is proposed. It is based on the minimal spanning tree (MST) [5] and takes advantage of its unique ability to rank multivariate data, preserve hierarchy and facilitate clustering. The proposed method can detect all the basic forms of edge structures and is suited for color or multispectral images of higher dimensions.

2. Vector Order Statistic
If we employ as a distance metric the aggregate distance is used as a distance metric.

\[ d_t = \sum_{i=1}^{n} \|X^i - X^0\|, \quad t = 1, 2, \ldots, n \]  

By ordering every for every vector in the set, we can have \( (d_{11} \leq d_{21} \leq \ldots \leq d_{n1}) \), which is in line with: \( X^{(1)} \leq X^{(2)} \leq \ldots \leq X^{(n)} \)

In this work a color image is viewed as a vector field [1], represented by a discrete vector valued function \( f(x) : \mathbb{Z}^d \rightarrow \mathbb{Z}^d \), where \( \mathbb{Z} \) represents the set of integers. For \( W \subseteq \mathbb{Z}^d \), \( x \in W \), \( t = 1, 2, \ldots, n \) is the size (number of pixels) of \( W \), \( f(x) \) will be denoted as \( X \). \( X^{(t)} \) will denote the ith ordered vector in the window according to the R-ordering method where the aggregate distance is used as a distance metric. Consequently, \( X^{(1)} \) is the vector median [3] in the window \( W \) and \( X^{(m)} \) is the outlier in the highest rank of the ordered vectors.
On this behalf, the base method for edge detection is Vector Ranking (VR), in which
\[ V_R = \|x^{(o)} - x^{(c)}\| \]  \hspace{1cm} (2)

3. Minimal Spanning Tree
A different approach is adopted here for ranking a set of observations from a vector-valued image. Using the MST, multivariate samples are ranked in such a way that the structure of the group is also made clear. Graph theory sketches the MST structure with the following definitions [5]. A graph is a structure for representing pairwise relationships among data. It consists of a set of nodes \( V = \{V_i\}_{i=1}^N \) and a set of links \( E = \{E_{ij}\}_{i \neq j} \) between nodes called edges. Applied in the description of a vector-valued image, it is represented by a graph \( G(V,E) \). Each node \( V_i \) corresponds to a pixel, while the undirected edge \( E_{ij} \) between two neighbor pixels \( (i, j) \) on the image grid has a scalar value equal to Euclidean distance of the corresponding vectors. A tree is a connected graph with no cycles. A spanning tree of a connected weighted graph \( G(V,E) \) is a connected subgraph of \( G(V,E) \) such that (i) it contains every node of \( G(V,E) \), and (ii) it does not contain any cycle. The MST is a spanning tree containing exactly \( (N - 1) \) edges, for which the sum of edge weights is minimum.

In what follows, the method is restricted for color RGB images \((i.e. \ p = 3)\) and in the case of a \( 3 \times 3 \) rectangular sliding window \( W \). However, the method is more general as it can be applied to higher dimensions and by using a window of an other size and/or shape. Given a set of \( N = 9 \) vectors corresponding to the pixels inside \( W \), the Euclidean MST (represented by \( T \)) is constructed in R3. Considering the edge types we would like to detect, three possible color distributions [3] can be usually found inside \( W \). If no edge is present and the central pixel is located at a uniform color region of the image, the distribution is unimodal denoting a “plain” pixel type. If there is an “edge” or “corner” point, a bimodal distribution is expected. Finally, in the case of a “junction”, pixels are expected to form three clusters. Thus, edges and corners are straight and angular boundaries of two regions, whereas junctions are boundaries of more than two regions.

4. System Design
In Minimal Spanning Tree method implementation, there are 3 main process, which are the calculation of the distances between neighboring pixels, finding the Minimal Spanning Tree route, and the deciding the output type (plain, edge, corner, or junction). Those three processes are done in a sliding window that the size is already defined, which is \( 3 \times 3 \). For that reason, the original input image matrix must be added with one pixel width of pixel on each side, so that the output of the pixels at the edge of the original image can be calculated.

The distances of neighboring pixel are calculated using Euclidean Distance. Then the Minimal Spanning Tree route can be determined using Kruskal Algorithm. \( T_1 \) variable was defined as a threshold parameter and \( T \) as total length to determine the pixel type. Next is the pixel type determination algorithm.

The process of the proposed method is summarized within the following steps:

- Construct the MST.
- Sort the derived MST-edges \( E_1,E_2, \ldots, E_8 \) in ascending order.
- Denote as “\( T \)” the total length of the MST.
- Define threshold parameter “\( T_1 \)” so that \( 0<T_1<1 \).
- If \( (E/7\times E_8, T_1) \) then unimodality exists (one distribution) → plain pattern.
  → \( R_1 = \text{mean}(T) \) is the detector’s output.
- Else if \( (E_8/E_7, T_1) \) then bimodality exist (two distributions)
  → edge or corner pattern.
  → Cut the maximum edge \( E_8 \) (two subtrees are generated, thus two separate clusters).
  → Find the mean value of the two clusters \( C_1 \) and \( C_2 \).
  → \( R_2 = \|C_1 - C_2\| \) is the detector’s output.
- Else, multimodality exist (three distributions) → junction pattern.
  → Cut the two bigger edges, \( E_7 \) and \( E_8 \) (three subtrees are generated, thus three separate clusters).
  → Find the mean value of each of the three clusters \( C_i, i = 1:3 \).
  → Compute the distance between the three cluster centers.
  \( R_{ij} = \|C_i - C_j\|, i, j = 1:3 \) for \( i \neq j \), \( i, j = 1:3 \) for \( i = j \).
  → \( R_3 = \text{mean}(R_{ij}) \) is the detector’s output.

5. Experiment Result
The images that used are lena.bmp, peppers.bmp, house.bmp, dan clown.bmp.

5.1 MST Method Test Using T1 Variation
The goal of this test is to prove whether changes in \( T_1 \) value affect the edge detection process.

Based on the result on Figure 1, \( T_1 \) value that gave best result is 1.0 on all samples images.

5.2 MST and VOS Comparison Test

5.2.1 Edge Quality Test
From the test result in Figure 2, it is seen that the result of edge detection using Minimal Spanning Tree Method gave edges that are more solid and not separated. Meanwhile using Vector Order Statistic gave
edges that are not solid and sometimes not connected between each other.

<table>
<thead>
<tr>
<th>T1 = 0.7</th>
<th>T1 = 0.8</th>
<th>T1 = 0.9</th>
<th>T1 = 1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
<tr>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
</tr>
<tr>
<td><img src="image9.png" alt="Image" /></td>
<td><img src="image10.png" alt="Image" /></td>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
</tr>
</tbody>
</table>

**Figure 1.** Result detection using MST with threshold variation
<table>
<thead>
<tr>
<th>Citra Masukan</th>
<th>Minimal Spanning Tree</th>
<th>Vector Order Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Image" /></td>
<td><img src="image2" alt="Image" /></td>
<td><img src="image3" alt="Image" /></td>
</tr>
<tr>
<td><img src="image4" alt="Image" /></td>
<td><img src="image5" alt="Image" /></td>
<td><img src="image6" alt="Image" /></td>
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<td><img src="image7" alt="Image" /></td>
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<td><img src="image12" alt="Image" /></td>
</tr>
</tbody>
</table>

Figure 2. Result comparison between MST and VOS based on edge quality
5.2.2 Algorithm’s Execution Time Test

<table>
<thead>
<tr>
<th>Citra Masukan</th>
<th>Minimal Spanning Tree</th>
<th>Vector Order Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elapsed time is</td>
<td>Elapsed time is</td>
<td></td>
</tr>
<tr>
<td>398.900481 seconds</td>
<td>14.334815 seconds</td>
<td></td>
</tr>
<tr>
<td><strong>6.6483 minutes.</strong> (MST = 100%)</td>
<td><strong>0.2389 minutes.</strong> (VOS = 3.59%)</td>
<td></td>
</tr>
<tr>
<td>Elapsed time is</td>
<td>Elapsed time is</td>
<td></td>
</tr>
<tr>
<td>416.205385 seconds</td>
<td>14.242192 seconds</td>
<td></td>
</tr>
<tr>
<td><strong>6.9368 minutes.</strong> (MST = 100%)</td>
<td><strong>0.2374 minutes.</strong> (VOS = 3.422%)</td>
<td></td>
</tr>
<tr>
<td>Elapsed time is</td>
<td>Elapsed time is</td>
<td></td>
</tr>
<tr>
<td>398.900481 seconds</td>
<td>14.334815 seconds</td>
<td></td>
</tr>
<tr>
<td><strong>6.6483 minutes.</strong> (MST = 100%)</td>
<td><strong>0.2389 minutes.</strong> (VOS = 3.549%)</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3. Result comparison and percentage between MST and VOS base on algorithm’s time execution.

From the result in Figure 3, it is clearly seen that Minimal Spanning Tree method takes longer time to finish than Vector Order Statistic method.

6. Result Evaluation

6.1 Best Threshold for MST Method

T1 value that gives best result on all sample images is 1.0.

The correlation between T1 and the detector output are as follow:

- If \( (E7/E8 \geq T1) \) then Result = plain
- else if \( (E6/E7 \geq T1) \) then Result = edge or corner
- else Result = junction

In this method, if E7 equal to E8 then it can be concluded that the pixel in observation is plain.

The pixel is considered edge or corner if E6 equal to E7 while E7 not equal to E8.

Last, if E6, E7 and E8 are all not equal, then the pixel can be considered as junction.

Data on Figure 3 was tested using T1 = 0.7 or T1 = 0.8, which gave plain type result, meanwhile if tested using T1 = 0.9 or T1 = 1.0, it was detected by the detector as junction.

From the experiments it is known that the increment of T1 value makes the sensitivity of the detector increases. Changes on T1 value can be used to get different detector sensitivity.

6.2 Edge Quality Comparison

Vector Order Statistic is a method that orders the sum of distances between pixels in a sliding window. A pixel has distance to all other pixels in the sliding window, including to itself. That distance is calculated using Euclidean distance equation.

In a uniform area, each vector will relatively close to each other and the distance value is smaller. Output value from this method is the average of the distances between pixels neighboring the pixel in question. Minimal spanning tree is a method that can rank data in a set into clear groups. Because of the nature of this method in which it only considers the neighboring pixel in the sliding window, the correlation between pixels is preserved. The point is, that the edge detection process put emphasis at the relationships.

Minimal Spanning Tree method also group pixels into clusters based on color similarities between neighboring pixels. By using the distances between each clusters as detector output, the edge resulted is finer.

6.3 Algorithm’s Execution Time Comparison

Based on the experiments, execution time needed for this MST method is longer than VOS method with average ratio of MST : VOS = 27 : 1. This is not a small ratio.

For the VOS, the method doesn’t do much iteration. All pixel type is processed using the same way, which is by using the defined equation.

Meanwhile for MST method, it is known that this method is doing a lot of iteration. All pixel type is processed using the same way, which is by using the defined equation.

7. Conclusion

From the experiments done, some conclusions found:

1. Minimal Spanning Tree method gave a more solid line than edge result from Vector Order Statistic method.
2. With the use of threshold parameter, detector sensitivity of Minimal Spanning Tree method can be defined according to the preferred result. Threshold value for best edge detection on Minimal Spanning Tree method is 1.
3. Minimal spanning Tree needs longer execution time than Vector Order Statistic with average ratio for sample images between MST and VOS is 100% : 3.733%.
4. Edge detection result from Minimal Spanning Tree method for images that have more detail will give
sharper edge than Vector Order Statistic.

8. **Suggestions**

   It is suggested to optimize the edge detection algorithm using minimal spanning tree, to shorten the execution time.

9. **References**


