ON APPROACHES FOR REPAIRABLE SYSTEM ANALYSIS
Renewal Process, Nonhomogenous Poisson Process, General Renewal Process

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Abstrak : *Terdapat beberapa strategi pemodelan pada repairable system, yaitu renewal process, nonhomogenous Poisson process, dan general renewal process. Model renewal process (RP) menggunakan asumsi perbaikan as good as new sementara model nonhomogenous Poisson process (NHPP) menggunakan asumsi as bad as old sedangkan general process menggunakan asumsi bahwa perbaikan akan mengembalikan kondisi operasi sistem dalam state antara as goos as new dan as bad as old. Makalah ini akan membahas perbandingan hasil pendekatan-pendekatan tersebut pada data real berdasarkan nilai likelihood. Hasil perbandingan menunjukkan pendekatan general renewal process lebih fleksibel untuk menangkap fenomena perbaikan yang menghasikan keadaan antara as good as new dan as bad as old.*
Kata kunci : *proses renewal, proses Poisson nonhomogenous, proses renewal general, nilai likelihood.*

Abstract : *There are some strategic modelling for repairable system, such as renewal process, nonhomogenous Poisson process, and general renewal process. Renewal process model use as good as new assumption, nonhomogenous Poisson process model use as bad as old assumption, and general renewal process assume that corrective maintenance will make operating system between as good as new and as bad as old. Based on real data, this paper will compare likelihood value of these model to find out the best model for repairable system modelling. The result show that general renewal process is more powerful to model maintenance that make condition between as good as new and as bad as old.*
Key words : *renewal processes, nonhomogenous Poisson process, general renewal process, likelihood value.*

1. Introduction

There are two major categories for repairable system, corrective maintenance and preventive maintenance (Mettas dan Zhao, 2005). Each category can be classified in the terms of the degree to which the operating condition of an item is restored by maintenance in the following way,

a) Perfect repair or maintenance: a maintenance action that restores the system operating condition to be "as good as new."

b) Minimal repair or maintenance: a maintenance action that restores the system operating state to be "as bad as old."

c) Imperfect repair or maintenance: a maintenance action that restores the system operating state to be somewhere between as good as new and as bad as old.

d) Worse repair or maintenance: a maintenance action that makes the operating condition worse than that just prior to failure.

e) Worst repair or maintenance: a maintenance action that makes the system fail or break down undeliberately.

The most common model for modelling time of repairable system is renewal processes (RP), include homogeneous Poisson process (HPP), and nonhomogenous Poisson process (NHPP). The models aren't always suitable to describe age of repairable system because the assumption that used is not realistics. Renewal process (RP) model use as good as new assumption, while nonhomogenous Poisson process (NHPP) use as bad as old assumption. In the application, both assumption have limitation and disagree with the real condition. Many maintenance activities aren't result to both extreme condition, but are between them (Gupta, et al., 2005). However, most probably one repairable system is impossible to be restore as good as new. Imperfect maintenance is concept where maintenance activities aren't restore system to the as good as new condition, but better than as bad as old. One of suitable model for this assumption is general renewal process (GRP) that is introduced by Kijima (Mettas dan Zhao, 2005).

Topic of this paper is about the use of general renewal process for repairable system analysis, and compare the result with renewal process and nonhomogenous Poisson process. The comparison use failure time data of

bottomer machine which is used for cement bag production. The comparison will show that general renewal process is more powerful than renewal process and nonhomogenous Poisson process in capturing corrective and preventive phenomenon.

Basically general renewal process model is considering condition between as good as new dan as bad as old. In this approach, hazard rate of intensity function is modeled by virtual age distribution. Based on *virtual* age distribution can be gained hazard rate of intensity function and the level of maintenance. Some researchers who develop these model are Dagpunar (1997), Jack (1997), Wang and Pham (1999), Guo, others (2000), Yanez, others (2002), Mettas and Zhao (2004), Mettas and Zhao (2005), and Wang and Pham (2008). The estimation of maintenance level is used lifetime virtual distribution concept based on general renewal process (Kijima and Sumita, 1986 ; Kijima, 1989). In general renewal process, continuity failure time form decreasing renewal quasi process and continuity maintenance time form increasing renewal quasi process. So, hazard rate system with imperfect repair will be gained by the use of renewal quasi process. Parameter estimation in hazard rate model can be solve by combination of Monte Carlo simulation technique and numerical analysis (Kaminsky and Kritsov, 1999; Yanez et al, 2002).

2. Model and Parameter Estimation
2.1. Renewal Process and Homogeneous Poisson Process

If a system is repairable and repair will restore system to condition as good as new for each repairing, then failure process is called renewal process. For renewal process, the times between failures are independent and identically distributed. The special case of renewal process is homogeneous Poison process (HPP), which has independent and exponential times between failures. Counting process is homogeneous Poisson process with parameter $\lambda > 0$ if :
- $N(0) = 0$ (initial condition)
- Independent increment, if number of failure in two disjoint interval time is independent.

- Independent time, for arbitrary k, $P[N(t, t+\Delta t)] = k$ only dependent to Δt.
- stationary increment, means the number of failure in any interval time is independent with initial condition.
- $P[N(\Delta t) = 1] = \lambda \Delta t + 0(\Delta t)$
- $P[N(\Delta t) \geq 2] = 0$
- The number of failure time in any interval time is Poisson distributed with parameter λt.

Implication of Poisson process definition is the number of event in interval (t_1, t_2) will be Poisson distribution with parameter $\lambda(t_2 - t_1)$. So, probability mass function (pmf) is

$$P\left[N(t_2) - N(t_1) = n\right] = \frac{\left[\lambda(t_2 - t_1)\right]^n e^{-\lambda(t_2 - t_1)}}{n!},$$
$$n = 0,1,2\ldots\ldots \quad (1)$$

Expectation of failure number in time *t* is
$$\Lambda(t) = E[N(t)] = \lambda t \quad (2)$$
which λ is called failure intensity or rate occurrence of failure (*ROCOF*). Intensity function is
$$u(t) = \Lambda(t) = \lambda \quad (3)$$
If t_1, $t_2,\ldots,t_n$ is independent, identically distributed exponentially random variable, then $N(t)$ correspondence with Poisson process. The parameter λ can be estimated by maximum likelihood estimation and estimator for λ is (Kulkarni,1999)

$$\hat{\lambda} = \frac{n}{\sum_{i=1}^{n} t_i} \quad (4)$$

2.2. Nonhomogeneous Poisson Process

If a repair restore system to as bad as old condition following each failure, then failure process is called nonhomogeneous Poisson process. NHPP definition is like homogenous Poisson definition, but its hazard rate λ is not constant. The cumulative number of failure up to time *t*, $N(t)$, can be modeled by NHPP. For counting process $\{N(t),\ t > 0\}$ modeled by NHPP, $N(t)$ follows Poisson distribution with $\Lambda(t)$. The probability that $N(t)$ is a given integer *n* is expressed by:

$$P\{N(t) = n\} = \frac{[\Lambda(t)]^n}{n!} e^{-\Lambda(t)}, n = 0,1,2\ldots (5)$$

$\Lambda(t)$ is mean value function. The function $\Lambda(t)$ describe expectation of cumulative number of failure. The underlying assumptions of the NHPP are:

- $N(0) = 0$
- $\{N(t), t \geq 0\}$ has independent increments
- $P\{N(t+h) - N(t) = 1\} = u(t) + o(h)$
- $P\{N(t+h) - N(t) \geq 2\} = o(h)$

$o(h)$ denotes a quantity that tends to zero for small h. Function $u(t)$ is intensity function. If $u(t)$ is known, then mean value function $\Lambda(t) = E[N(t)]$ is given by

$$\Lambda(t) = \int_0^t u(s)\, ds \qquad (6)$$

Inversely, knowing $\Lambda(t)$, the failure intensity at time t can be obtain by,

$$u(t) = \frac{d\Lambda(t)}{dt} \qquad (7)$$

So, the probability of exactly n events occurring in the interval $(a, b]$ is given by

$$P\left[N(b) - N(a) = n\right] = \frac{\left[\int_a^b u(t)\, dt\right]^n e^{-\int_a^b u(t)\, dt}}{n!}, n = 0,1,2..... \qquad (8)$$

One of the most of the failure intensity used in reliability analysis of repairable systems, the Crow (*Army material System Analysis Activity/AMSAA*) model, is as follows

$$u(t) = \lambda\beta t^{\beta-1}$$
$$E[N(t)] = \lambda t^{\beta} \qquad (9)$$

where,
$N(t)$: number of observed failure in interval $(0, t)$
$u(t)$: failure intensity function
λ, β : model parameter ($\lambda > 0$, $\beta > 0$)

A nonhomogeneous Poisson process with an intensity function represents the rate of failures or repairs. NHPP can model a system that is improving, deteriorating, or remaining stable. The λ parameter is scale parameter, indicate the dispersion of failure time. The β parameter is shape parameter and its value depends on behavior process, whether system is improving, deteriorating, or remaining stable,

- If $0 < \beta < 1$, the failure/repair rate is decreasing. Thus, your system is improving over time.
- If $\beta = 1$, the failure/repair rate is constant. Thus, your system is remaining stable over time.
- If $\beta > 1$, the failure/repair rate is increasing. Thus, your system is deteriorating over time.

Parameter Estimation for NHPP Model

Parameter estimation of NHPP model must consider type of observation, failure truncated or time truncated. Process is said failure truncated if it is observed until the fixed number of failure have occurred. If failure time follows Weibull process with shape parameter β and scale parameter λ and data is truncated at n^{th} failure with $0 < t_1 < t_2 < ... < t_n$ denoting successive failure time, then the probability density and likelihood function is

$$f(t) = \lambda\beta(\lambda t)^{\beta-1}\exp\left[-(\lambda t)^{\beta}\right]$$

$$L(t_1,\ t_{2,}....t_n; \lambda, \beta) = \lambda^n\beta^n\exp\left(-\lambda t_n^{\beta}\right)\prod_{i=1}^{n} t_i^{\beta-1} \quad (10)$$

Maximum *likelihood* estimation for β and λ are

$$\hat{\beta} = \frac{n}{\sum_{i=1}^{n-1}\ln(t_n/t_i)} \qquad (11)$$

$$\hat{\lambda} = \frac{n}{t_n^{\beta}} \qquad (12)$$

where

n : number of failure
t_n : n^{th} failure time

A process is said to be time truncated if it is observed for a fixed length of time. If failure time follow Weibull process with shape parameter β and scale parameter λ and data is truncated at time T with $0 < t_1 < t_2 < ... < t_n$ denoting successive failure time, then likelihood function is given by

$$L(t_1, t_2, ..., t_n, \lambda, \beta) = \lambda^n\beta^n\exp\left(-\lambda T^{\beta}\right)\prod_{i=1}^{n} t_i^{\beta-1} \quad (13)$$

Maximum likelihood estimatior for β and λ are

$$\hat{\beta} = \frac{n}{\sum_{i=1}^{n}\ln(T/t_i)} \qquad (14)$$

$$\hat{\lambda} = \frac{n}{T^{\hat{\beta}}} \qquad (15)$$

where

n	: number of failure	
T	: end time of observation	
t_i	: ith failure time	

2.3. General Renewal Process (GRP)

So far, in RP or NHPP we just have choice as good as new or as bad as old for modelling impact of repair. How if the real condition is between both condition? GRP model is more powerful for *repairable system* modeling because its flexibility with various maintenance assumption. It is caused GRP has parameter indicate phenomenon of maintenance, as good as new, as bad as old or imperfect repair. There are two type of GRP, Kijima Model I (Kijima and Sumita, 1986) and Kijima Model II (Kijima, 1989).

Kijima Model I

Kijima I Model is imperfect repair model which uses virtual age of a repairable system. If a system has virtual age $V_{n-i} = y$ immediately after $(n-1)$th repair, the nth failure time X, is assumed to have distribution function :

$$\Pr\left(X_n < X / V_{n-1} = y\right) = \frac{F(X+Y) - F(y)}{1 - F(Y)} \qquad (16)$$

Where $F(X)$ is distribution function of the time to failure of a new system. The real age of the system is $S_n = \sum_{i=1}^{n} X_i$. Let q is degree of nth repair where $0 \leq q \leq 1$ and the virtual age of new system is $V_0 = 0$. Kijima I model is constructed based on assumption that n^{th} repair can not eliminate the damage incurred before $(n-1)^{th}$ repair. It will reduce additional age X_n to qX_n where $0 \leq q \leq 1$. So, virtual age after repair n^{th} is

$$V_n = V_{n-1} + qX_n \qquad (17)$$

$$V_n = q\left(X_1 + X_2 + ... + X_n\right) \qquad (18)$$

The expected number of failure in interval [0, *t*] can gained from *general renewal equation by* Kijima and Sumita (1986).

$$H(t) = \int_0^t \left[g(\tau|0) + \int_0^r h(x)g(\tau - x|x)dx \right]d\tau \qquad (19)$$

where

$$g(\tau|x) = \frac{f(t+qx)}{1 - F(qx)}, t, x, \geq 0 \qquad (20)$$

It is impossible to obtain close form solution of (19). Alternatively, Kaminskiy dan Krivtsov (1998) proposed Monte Carlo simulation to find out solution. Let a independent and identical sample from *repairable system* be observed at discrete time interval over period [0, *t*], then empirical cumulative intensity function $\Lambda_e (t)$ can be estimated at the end interval t_i, $i = 1, 2, .. , n$

$$\Lambda_e(t) = \frac{1}{k}\sum_{j-1}^{k} N_j(t_i) \qquad (21)$$

Where $N_j(t_i)$ is failure number of jth system in interval [0, t_1] and k is the number of system at time $t = 0$. Monte Carlo simulation generates cumulative intensity function $\Lambda_{mc}\left(F(\alpha_1, \alpha_2, ...\alpha_n, \tau), q, t\right)$. Thus, the solution

$$\min_{\alpha_1, \alpha_2, ...\alpha_n, q}\left(\sum_{i=1}^{[t/n]}\left(\Lambda_e(t_i) - \Lambda_{mc}\left(F(\alpha_1, \alpha_2, ...\alpha_n, \tau), q, t_i\right)\right)^2\right)$$

provide non linear least square estimation $\alpha_1, \alpha_2, ...\alpha_n$ dan q.

Kijima Model II

Consider repairable system, observed from time $t = 0$. Denote by t_1, t_2,... are the successive failure time and inter failure time denoted by x_1, x_2,... . Thus, we have

$$x_i = t_i - t_{i-1} \qquad t = 1, 2, ... \qquad (22)$$

where $t_0 = 0$.

Kijima II model is extension of Kijima I model. The different between them is on assumption about impact of repairing on the damaged incurred. Kijima I model assume the nth repair can remove the damage incurred only during the time between the $(n-1)^{th}$ and n^{th} failures. In practice, not only does the nth repair depend on $(n-1)^{th}$ repair, but also it depends on all previous repair. We assume that the repair action could remove all damage accumulated up

to n^{th} failure, accordingly, the virtual age after the nth repair become

$$V_n = q(V_{n-1}) + x_n \qquad (23)$$

where $0 \le q \le 1$, indicate the degree of nth repair. As a result from (23),

$$V_0 = 0$$

$$V_1 = qx_1$$

$$V_2 = q(qx_1 + x_2)$$

.

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$$V_n = q(q^{n-1}x_1 + q^{n-2}x_2 + .. + x_n) \qquad (24)$$

If a system have virtual age $V_{n-1} = y$ immediately after $(n\text{-}1)^{th}$ repair, then n^{th} failure time X follows the cumulative distribution function (cdf).

$$F(X|V_{n-1} = y) = \frac{F(X+Y) - F(Y)}{1 - F(Y)} \qquad (25)$$

Clearly, $q = 0$ corresponds to a perfect repair (RP, as good as new) while $q = 1$ leads to a minimal repair (NHPP, as bad as old). The case of $0<q<1$ corresponds to an imperfect repair (better than old but worse than new) while $q >1$ leads to worse or worst repair (worse than old). The case of $q < 0$ suggested a system restored to a condition of better than new. This is also shown that GRP have flexibility for modeling impact of repair or maintenance.

Quantity of q can be an index for repair effectiveness. Index q can be used to compute restoration factor (RF)

$$RF = 1 - q \qquad (27)$$

Restoration can describe age of the system after repair. Restoration factor 100% indicate the system can operate like the new condition after repair. This imply that age of the system is zero when it reactivate. Restoration factor 0 indicate that the system reactivate like condition when it fail and this imply that age of the system equal to age of the system when it fail. The other value, for example 0.25, indicate age of the system when it reactivate after repair equal to 75% when it fail. In this case, restoration factor is inverse of effectiveness of maintenance.

Maximum Likelihood for Parameter Estimation

Monte Carlo approach by Kaminskiy and Kivtsov (1998) used simulation technique for GRP parameter estimation. It need estimation of *time to first failure* (TTFF) distribution from large sample, so need much time to estimate model parameter. For this reason, *maximum likelihood estimation* (MLE) is more considered to estimate parameter of GRP model.

MLE can be considered to be used if there are enough data available. If t_1, t_2,...t_n inter failure time $(i–1)^{th}$ and i^{th} is assumed Weibull distributed, then t_i failure time is distributed accordingly following *CDF*

$$F(t_{i-1} = v_{i-1}) = \frac{F(x_i + v_{i-1}) - F(v_{i-1})}{1 - F(v_{i-1})}$$

$$= \frac{e^{-\lambda v_{i-1}^{\beta}} - e^{-\lambda(x_i + v_{i-1})\beta}}{e^{-\lambda v_{i-1}^{\beta}}} \qquad (26)$$

$$= 1 - e^{-\lambda\left[(x_i + v_{i-1})^{\chi} - v_{i-1}\right]}$$

Thus, conditional probability density function t_i is

$$f(t_i|t_{i-1}, t_i, ..., t_1) = f(t_i|t_{i-1})$$

$$= \lambda\beta(x_i + v_{i-1})^{\beta-1} \exp\left[-\lambda(x_i + v_{i-1})^{\beta} - v_i^{\beta}\right] \qquad (24)$$

where $t_i > t_{i-1}$

So, the likelihood function is

$$L\{data|\lambda, \beta, q\} = f(t_i)f(t_2|t_1)...f(t_n|t_{n-1})R(T|t_n)$$

$$= \lambda^n \beta^n \left[e^{-\lambda\left[(T-t_n+v_n)^{\beta} - v_n^{\beta}\right]}\right]^{\delta} \cdot$$

$$\prod_{i=1}^{n}\left[(x_i + v_{i-1})^{\beta-1} e^{-\lambda\left[(x_1 - v_{i-1})^{\beta} - v_{i-1}^{\beta}\right]}\right]$$

$$\qquad (25)$$

where

$$\delta = \begin{cases} 0 & \text{if the test is failure truncated} \\ 1 & \text{if the test is time truncated} \end{cases}$$

Taking the natural log on both sides:

$$\log L\{data|\lambda, \beta, q\} = n(\ln\lambda + \ln\beta) +$$

$$- \lambda\delta\left[(T - t_n + v_n)^{\beta} - v_n^{\beta}\right] - \lambda\sum_{i=1}^{n}\left[(x_i + v_{i-1})^{\beta} - v_{i-1}^{\beta}\right] +$$

$$(\beta - 1)\sum_{i=1}^{n}\ln(x_i + v_{i-1}) \qquad (26)$$

where v_i can be obtained from equation (26).

Basically, we can obtain parameter λ, β, and q that maximize (25) or (26) with differentiate the equation to be each parameter and set the differentiation equal to zero. Unfortunately, there is no close form mathematical solution for this equation. Thus, a numerical algorithm has been developed to solve this equation likelihood such as the Newton search, genetic algorithms, annealing method, etc. The problem was approached with a form of the Newton search method, which is closely related to the Quasi-Newton method.

3. Application to real data

For applied of methods will be used failure time of tuber machine that is used by cement packaging manufacturing to make cement bag. This machine consist of paper roll stand, web brake, edge position controller, web drawing unit, perforating unit, longitudinal pasting unit, tube forming unit, tear off unit, stacking conveyor, and printer (Hadiyatul, 2006). The data set is presented in Table 1. There are 50 failure time data. The first failure was recorded at time 1.58. The second failure was recorded at time 1.83. The last failure was occurred at 407.4. This failure data is failure truncated data. Based on this data, the previous models will be used for modelling the failure number of this data. The models are renewal process, nonhomogeneous Poisson process, GRP I (Kijima I) and GRP II (Kijima II). Each model has three parameter, λ, β, q and generally they are obtained by maximum likelihood methods. The criteria for choosing the best model is likelihood value of the model. The best model is model which has the largest likelihood value.

Tabel 1. Failure time data of Tuber machine (hours)

1.58	82.58	163.99	257.75	353.58
1.83	90.91	169.74	269.00	356.41
13.75	93.74	170.99	283.08	363.49
21.08	115.92	172.99	284.91	364.74
34.83	123.04	182.07	289.66	370.07
41.50	130.66	187.82	297.99	377.40
44.50	130.89	204.49	298.66	389.08
66.75	139.76	213.74	305.74	396.90
67.42	153.91	241.41	337.74	398.40
68.75	160.91	241.58	348.91	407.98

Table 2 shows the result of parameter estimates. For RP column, it is assumed the repair activities are perfect repairs and the failure intensity is as good as new. $\hat{\lambda}$ can be calculated by equation (4). For NHPP column, it is assumed that the repair actions restore the system operating state to be as bad as old. Therefore $\hat{\beta}$ and $\hat{\lambda}$ are calculated by equation (11) and (12). For GRP model Kijima I and Kijima II, as a mentioned above, the solution can not be obtain analytically, so numerical methods is applied by Weibul ++7 software.

Table 2. The parameter value of λ, β, and q

Parameter	RP	NHPP	Kijima I	Kijima II
$\hat{\beta}$	1.0000	0.9174	1.0997	1.2107
$\hat{\lambda}$	0.1226	0.1974	0.0939	0.0652
$\hat{q}$	0.0000	1.0000	0.0023	0.2003
LK value	-154.9939	-152.6653	-152.5241	-152.2435

It can be seen in Table 2, because the likelihood value of Kijima II is minimum, so the best model for this data set is Kijima II. But, Kijima model II is not always the best model for another case. It is depend on failure data characteristics. This result only justify that Kijima model II can detect imperfect repair or maintenance that restores the system operating state to be somewhere between as good as new and as bad as old, which can not detected by renewal process or nonhomogenous Poisson Process.

4. Conclusion

Repairable system analysis can be approach by renewal process, nonhomogenous Poisson Process, and general renewal process. Each model has different assumption. Benefit of general renewal process compare to other model is about its ability to capture repair that restores the system operating between as bad as old and as good as new. It has shown by analytically and numerically. In practice, general renewal process has limitation in problem due to estimate the parameter, because no close form solution from likelihood equation. To overcome (solve) this problem can be applied numerical methods.

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