ARIMA Modeling of Tropical Rain Attenuation on a Short 28-GHz Terrestrial Link

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Abstract—This letter reports the result of modeling of the tropical rain attenuation at 28 GHz adopting the auto-regressive integrated moving average (ARIMA) model. The result obtained is useful for the evaluation of transmission system design in radio communications at millimeter-wave frequencies in tropical areas. In this research, radio power measurement on a 56.4-m link at 28 GHz was carried out in Surabaya, Indonesia, with a data acquisition system that recorded a sample once every second. Approximation of the data by ARIMA($p$, $d$, $q$) models for every rain event was carried out in order to obtain a valid time series model. In the validation, comparisons were made of the distributions of attenuation from the models against those from direct measurement of attenuation and from estimation based on the synthetic storm method. Comparisons were also made of the attenuation slope distributions against that from the measurement. Each of rain attenuation time series obtained from events in February 2009 was found to be well approached by the ARIMA model with various sets of parameters ($p$, $d$, $q$). The best model was found to be ARIMA(0, 1, 1), as indicated by the Akaike Information Criteria (AIC) and Schwarz Bayesian Criteria (SBC) test results. A procedure for generating rain events based on the model is presented.

Index Terms—auto-regressive integrated moving average (ARIMA) model, millimeter-wave, rain attenuation, terrestrial radio.

I. INTRODUCTION

Radio waves above 10 GHz are highly influenced by rain-induced attenuation, not just the magnitude thereof, but also the rate of its variation. In order to achieve a reliable design of wireless communication systems operating at these frequencies, it is necessary to evaluate the design in a simulation against time series of rain attenuation generated from a realistic model [1], [2].

Models based on auto-regressive moving average (ARMA) processes have been used to model rainfall rates [3], [4]. The ARMA model can only be used for stationary time series data and hence cannot be used for nonstationary rain events. Therefore, a step is required to convert the nonstationary data into stationary ones, which is available in auto-regressive integrated moving average (ARIMA) in the form of data differencing. The ARIMA model has been explored in [5] to model rainfall rate data recorded in a single event only. With regards to rain attenuation, similar efforts have been done, such as one reported in [6] based on estimates of rain attenuation obtained from applying the synthetic storm technique (SST) to rainfall rate measurements. To the best of our knowledge, the ARIMA model previously applied to rainfall rate measurements and rain attenuation estimates from SST has never been used for modeling of rain attenuation directly measured on a real radio link. The study reported herein concerns ARIMA modeling of rain attenuation based on data recorded in multiple events and elaborates on the data generation based on the model.

In order to measure the rain attenuation, a terrestrial radio communication system working in the frequency of 28 GHz was installed in the campus area of the Institut Teknologi Sepuluh Nopember (ITS) in Surabaya, Indonesia. The procedure for ARIMA modeling was further applied on the rain attenuation data. The resulting model was then tested by comparing the distributions of attenuation and of attenuation gradients (slopes) to those obtained from measurement. A comparison was also made of the distribution of attenuation to that resulting from the SST [6] employing the power-law coefficients suggested in the ITU-R P.838-3 [7]. Finally, a procedure of event generation based on the ARIMA modeling result is described.

Section II of this article describes the methodology, detailing the measurement system and the ARIMA modeling. Section III explains the results of measurement and analysis, while the final section presents the conclusion.
II. METHODOLOGY

A. Measurement System

The measurement of rain attenuation at 28 GHz was carried out on the rooftop of the Electrical Engineering Department building in the ITS campus area in Surabaya (112°43’ E, 7°13’ S), which might typify a city in the region of maritime tropical climate. A block diagram of the system is given in Fig. 1 [8]. The distance between transmitter and receiver is 56.4 m, yielding a free-space loss of 96.4 dB. The first Fresnel zone was assured to be clear of potential obstacles. The received power data acquisition at the receiver subsystem was done after down conversion to 10 MHz, with sampling period of 1 s. A disdrometer-based measurement of rainfall rate was also carried out in parallel at the same sampling rate to allow rain attenuation estimation by means of the SST, using wind velocity data from the local weather office [6].

B. ARIMA Modeling

In ARMA modeling, $X_t$ is an ARMA($p,q$) process if it is stationary and if for every $t$

$$\Phi_p(B)X_t = \Theta_q(B)\alpha_t$$

(1)

with an AR operator

$$\Phi_p(B) = 1 - \phi_1 B - \cdots - \phi_p B^p$$

(2)

and a MA operator

$$\Theta_q(B) = 1 - \theta_1 B - \cdots - \theta_q B^q$$

(3)

where $\phi_m$ represents the $m$-th AR coefficient, $\theta_n$ the $n$th MA coefficient, $X_t$ rain attenuation, $\alpha_t$ the error, and $B$ backward shift operator [9], [10].

A process $X_t$ is said to follow the integral ARMA model, which is referred to as ARIMA($p,d,q$), if it can be written as

$$\Phi_p(B)(1 - B)^d X_t = \theta_0 + \Theta_q(B)\alpha_t$$

(4)

where $d$ is a nonnegative integer. Equation (4) reduces to the ARMA($p,q$) when $d = 0$.

The Box–Jenkins procedure is a widely used one for ARIMA modeling. A summary of the steps in the ARIMA modeling using this procedure is as follows [9]. First, the Box–Cox transformation $T(\cdot)$ is used to determine the data variance stationarity, which is done by estimating the value of parameter $\lambda$ of the original data $X_t$

$$T(X_t) = \frac{X_t^\lambda - 1}{\lambda}.$$  

(5)

If $\lambda = 1$, the original data are already stationary in the variance, while $\lambda \neq 1$ means otherwise, which necessitates transformation into data that are stationary in the variance. The value $\lambda = 0$ means the data should be transformed by $\ln(X_t), \lambda = 0.5$ means transformation by $[X_t]^{0.5}$, while $\lambda = -0.5$ means transformation by $1/[X_t]^{0.5}$. If $\lambda$ is not equal to any of the above values, then rounded values are used.

Next, a test for the stationarity in data mean is conducted by visual inspection of the auto-correlation function (ACF). If the ACF shows a slow downward pattern, the data are not yet stationary in the mean, hence differencing is required to achieve the stationarity in the mean. Conversely, if it shows a fast downward pattern, the data are already stationary in the mean.

The following step involves identification of the order of the ARMA model using the ACF and partial auto-correlation function (PACF) of the stationary data. The parameter values of the obtained models are then estimated and subsequently tested with regard to the $p$-values of those coefficients [9]. If the $p$-values of the constants and coefficients are less than 0.05, then the constants and coefficients are statistically significant and valid for application. Otherwise, they are eliminated from the model.

The next step is a diagnostic check, in which the residuals of the model were tested for their fulfillment of requirements for appropriateness in the ARIMA model. It is required that the residuals should be white and normally distributed. Evaluation of the whiteness was carried out using the Ljung–Box test, which determines that they are white if their $p$-value (not to confuse with the order $p$ of the AR segment of the ARIMA) is greater than 0.05. Next was residual normality diagnosis by the Kolmogorov–Smirnov test [11]. If the $p$-value is greater than 0.05, the residuals are said to be normally distributed. There might be several appropriate sets of parameters ($p,d,q$) with different mean squared error (MSE) values. The best set was determined by means of the Akaike Information Criteria (AIC) and Schwarz Bayesian Criteria (SBC) tests [9], [10].

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**TABLE I
RESULTS OF ARIMA MODELING OF RAIN ATTENUATION**

<table>
<thead>
<tr>
<th>Date of Event</th>
<th>No. of Samples</th>
<th>Event Prob. P(X)</th>
<th>ARIMA Model Parameters</th>
<th>Cumulative Prob. of Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>February 10</td>
<td>722</td>
<td>0.17</td>
<td>(0,1,1)</td>
<td>0.17</td>
</tr>
<tr>
<td>February 14</td>
<td>549</td>
<td>0.14</td>
<td>(0,1,1)</td>
<td>0.31</td>
</tr>
<tr>
<td>February 21</td>
<td>693</td>
<td>0.17</td>
<td>(3,0,0)</td>
<td>0.48</td>
</tr>
<tr>
<td>February 22</td>
<td>636</td>
<td>0.15</td>
<td>(5,1,0)</td>
<td>0.63</td>
</tr>
<tr>
<td>February 24</td>
<td>1133</td>
<td>0.27</td>
<td>(0,1,1)</td>
<td>0.90</td>
</tr>
<tr>
<td>February 25</td>
<td>432</td>
<td>0.10</td>
<td>(3,0,0)</td>
<td>1.00</td>
</tr>
<tr>
<td>Total</td>
<td>4165</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
III. MEASUREMENT RESULTS AND DATA ANALYSIS

The result of data processing for rain events that occurred between February 10–25, 2009, is summarized in Table I. The results of ARIMA modeling on every event are shown in the table. It shows that events on February 10, 14, and 24 each yielded an ARIMA model of only one appropriate set of parameters \((p, d, q)\). The event on February 21, 2009, could be approached by ARIMA\((3, 0, 0)\), ARIMA\((4, 1, 0)\) and ARIMA\((0, 1, 1)\); that on the 22nd could be approached by ARIMA\((5, 1, 0)\) and ARIMA\((0, 1, 1)\), whereas that on the 25th by ARIMA\((3, 0, 0)\) and ARIMA\((0, 1, 1)\). Data errors of the models can be used later to generate rain attenuation data from the model.

Model analysis and validation was carried out by generating time series of rain attenuation following the ARIMA model with the given set of parameters. The distribution was compared to those of the corresponding original measurements, as well as those of rain attenuation estimates obtained using the SST from rainfall rate measurements made simultaneously with the attenuation measurement. The rainfall rate measurements \(R\) were transformed into an equivalent rain attenuation \(A_R\) along a 56.4-m link using the power-law approximation,

\[
A_R = kR^n \times \frac{56.4}{1000}
\]

where parameters \(k\) and \(n\) depend on the radio wave frequency and polarization and the rain microphysical characteristics, which for the purpose of this study follow the values suggested in ITU-R P.838-3 [7].

Fig. 2 presents a comparison of complementary cumulative distribution functions (ccdfs) between the measurement, the SST result, and the prediction based on ITU-R Recommendation P.530-12 [12]. For rain attenuation less than 13.55 dB, probabilities of attenuation being exceeded from the SST results are smaller than those from the measurement. Both the SST and measurement exhibit larger attenuation than the ITU-R
prediction. The discrepancy is likely due to the relatively short period of our measurement.

Fig. 3(a)–(d) show the comparisons of ccdfs between the measured rain attenuation, the SST, and the data generated from the ARIMA models with parameters and coefficients found from the corresponding measurements. Distributions obtained from data generated using the four models are shown in the figure. Generally, distributions of the ARIMA-generated attenuation start to deviate far from those of the measurement for small exceedence probabilities, i.e., smaller than 0.1%.

Meanwhile, Fig. 4(a)–(d) present the comparisons of cdfs of attenuation slope between each of the modeling results and the corresponding measurements. The cdfs of attenuation slopes for the ARIMA(0, 1, 1) and the measurement are very close to each other, as shown in Fig. 4(a), more so than for the case of the other ARIMA models. This might indicate that ARIMA(0, 1, 1) can be safely used to model and generate rain attenuation, at least for the examination of outage probabilities down to 0.1%. In fact, with varying degrees of accuracy, ARIMA(0, 1, 1) can be used to model all of the recorded events as shown in Table I.

This is also supported by the finding that the ARIMA(0, 1, 1) model demonstrates the smallest results of the AIC and SBC tests.

In order to refine the ARIMA model, detection of outliers was applied [10], [11]. In processing the rain attenuation measurements of the six events listed in Table I, five outliers were detected. Although the results are not shown herein, it was found that the detection of outliers improves the results of the AIC and SBC tests. This reaffirms that the ARIMA(0, 1, 1) model exhibits the best results in the AIC and SBC tests.

It should be noted, however, that the ARIMA(0, 1, 1) modeling results for different events might yield their respective error variances and MA coefficients. Each of these possibilities must be considered in devising a procedure to generate rain events on computer for evaluation of radio communication systems. Accordingly, the steps suggested for generating rain events can be described as follows.

**Step I:** Generate values uniformly, i.e., following $U(0,1)$, a number $n_t$ of times, with $n_t$ in the same order of the desired number of events (e.g., the annual average number of events).
Every time they are generated, the values are checked against the groups in the last column (cumulative probability of occurrence) of Table I.

Step II: Generate rain attenuation time series in accordance with the ARIMA(0, 1, 1) with the corresponding MA coefficient and error variance, with number of samples equal to the number of times the set is selected in Step I multiplied by a typical number of samples in a real event, e.g., 1000.

IV. CONCLUSION

An ARIMA modeling has been developed from the measurements of tropical rain attenuation on a short 28-GHz terrestrial radio link. From the evaluation of the model, it can first be concluded that rain attenuation time series measured in a rain event can be approached by the ARIMA model with more than one possible set of parameter values \((p, d, q)\). Second, all measured rain events could be approached by ARIMA(0, 1, 1). Third, ARIMA (0, 1, 1) could be used to generate rain attenuation data for system evaluation as it yields distributions of attenuation and slope close to the measurement results for outage probabilities down to 0.1%. The AIC and SBC results suggest the adoption of ARIMA (0, 1, 1) for rain attenuation. Finally, steps required to generate rain attenuation based on the ARIMA model has accordingly been described. While the specific result we obtained might not be directly applicable to other locations or climatic regions, the modeling procedure described earlier provides a generic framework of ARIMA modeling of nonstationary rain attenuation.

REFERENCES


